Approved Reference Sheet

1. The length of the arc of $\mathbf{r}(t)$ between $\mathbf{r}(a)$ and $\mathbf{r}(b)$ is given by

$$\int_a^b |\mathbf{r}'(t)|\, dt.$$

2. The curvature of the curve $\mathbf{r}(t)$ is given by

$$\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

3. If z is a function of x and y, which in turn are functions of u and v, then

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

4. The directional derivative of $f(x, y, z)$ in the direction of the ***unit*** vector $\mathbf{u}$ is given by

$$\mathbb{D}_{\mathbf{u}} f(x, y, z) = \nabla f(x, y, z) \cdot \mathbf{u}.$$

5. The area of the part of the surface $z = f(x, y)$ which is the image under f of the region $D \subset \mathbb{R}^2$ is given by

$$\iint_D \sqrt{(f_x(x, y))^2 + (f_y(x, y))^2 + 1}\, dA.$$

6. The change of coordinates from spherical to rectangular is given by

$$\begin{aligned} x &= \rho \sin\varphi \cos\theta \\ y &= \rho \sin\varphi \sin\theta \\ z &= \rho \cos\varphi. \end{aligned}$$

7. Let E be the spherical wedge $\{(\rho, \theta, \varphi) : a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \varphi \leq d\}$. Then

$$\iiint_E f(x, y, z)\, dV = \int_c^d \int_\alpha^\beta \int_a^b f\Big(\rho \sin\varphi \cos\theta, \rho \sin\varphi \sin\theta, \rho \cos\varphi\Big) \rho^2 \sin\varphi\, d\rho\, d\theta\, d\varphi.$$

8. The Jacobian of the transformation $T(u, v) = \langle x(u, v), y(u, v) \rangle$ is defined

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

9.

$$\iint_{T(S)} f(x, y)\, dA = \iint_S f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| dA.$$

10. Let $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle$ be a vector field and let the curve C be the image of $\mathbf{r} : [a, b] \to \mathbb{R}^2$. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C P\ dx + Q\ dy = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\ dt.$$

11. Let C be a positively oriented, piecewise-smooth, simple, closed curve in the plane and let D be the region bounded by C. Then

$$\int_C P\ dx + Q\ dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)\ dA.$$