

Homework 6, Math 215

Prove or disprove the following statements

1. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, xy \neq 1$
2. $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, x^2 + y^2 + z^2 \leq 1$

Prove using the ϵ, N definition of limits that

1. $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$.
2. $\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1$.

Prove using the ϵ, δ definition of limits that

$$\lim_{x \rightarrow 0} x^2 = 0.$$