Homework 1 Solutions

Problem 1. *Write a truth table for*

a) *not* $(P$ *or* $Q)$
b) *not* $(P$ *and* $Q)$
c) P *or not* Q

Solution.

a)

P	Q	P or Q	not $(P$ or $Q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

b)

P	Q	P and Q	not $(P$ and $Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

c)

P	Q	not Q	P or not Q
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Problem 2. *Which of the following statements is the negation of the statement that there is a student who got a perfect score on every test in the class?*

a) *there is a student who got a 0 on every test,*
b) *all students got perfect scores on every test,*
c) *no student got a perfect score on every test,*
d) *every student missed some points on some test.*

Solution. If there is no such student, then this does not tell us anything about students that failed. Therefore, it cannot be (a). Furthermore, if there is no student that got a perfect score on all tests, then of course all the students did not get a perfect score on all tests, so (b) is out. Now, note that saying there is no student who got a perfect score on every test in the class is exactly the opposite of saying there is one. Therefore, (c) is the negation of the original statement. Finally, if there is no student who got perfect scores on all tests, then every student did not have a perfect score on every single test (possibly some tests, just not all); in other words, every student missed some points on some test. That is, (d). Hence, the answer is **(c) and (d)**.

Problem 3. *Which of the following implications is true?*

a) $n^2 + n - 2 = 0$ implies $n = -2$ or $n = 1$.
b) $n^2 + n - 2 = 0$ implies $n = -2$.
c) $n^2 + n - 2 = 0$ implies $n = 2$ and $n = 1$.

Solution. First, let's factor $n^2 + n - 2 = 0$. We get $(n+2)(n-1) = 0$. If we multiply two numbers together and get 0, one of the numbers must be 0. Therefore, $(n+2)(n-1) = 0$ means either $n + 2 = 0$ or $n - 1 = 0$. In other words, either $n = 0 - 2 = -2$ or $n = 0 + 1 = 1$. That is, $n^2 + n - 2 = 0$ implies $n = -2$ or $n = 1$, so **(a) is true**. Notice (b) is false, because n could be 1, which would make the polynomial equal to 0, but would not imply $n = -2$. Also, (c) is false because $n = 2$ and $n = 1$ cannot both be true (then $2 = n = 1$ and if $2 = 1$ we know something's wrong).