

Sample Final Math 215

- What is the negation of the statement that today it will be sunny and hot?
- Prove by induction that

$$\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

- Prove that

$$\lim_{x \rightarrow 0} x^2 = 0.$$

- Write the contrapositive of the statement: if  $n^2$  is an odd integer then  $n$  is an odd integer. Now prove the contrapositive
- Prove by induction that if  $|X| = k$  then the power set  $P(X)$  has  $2^k$  elements.
- Prove that  $A \subset B$  if and only if  $A \cap B = A$ .
- For each of the following decide if the statement is true or false. If true prove it. If false, state the negative and prove the negative
  - a)  $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} : x + y = -1$ .
  - b)  $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} : x^2 - y^2 \leq 1$ .
  - c)  $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, \exists z \in \mathbb{R} : x^2 + y^2 + z^2 = 1$ .
- Prove that  $\lim_{n \rightarrow \infty} \frac{1}{n^{3/2}} = 0$ .
- Suppose  $f : X \rightarrow Y$  is a surjection. Construct an injection  $g : Y \rightarrow X$ . Now show that if  $X, Y$  are finite sets with  $|X| < |Y|$  then there can be no surjection from  $X$  to  $Y$ .
- Prove that  $\sqrt{2}$  is irrational
- Prove that the union of two denumerable sets is denumerable and then prove by contradiction that if a set  $X$  is uncountable and a subset  $A$  is denumerable then  $X \setminus A$  is uncountable.

- a) How many ways are there of forming a group of 4 people out of total group of 6 people if we do not pay attention to the order in which they are chosen?
- b) How many ways are there if we do pay attention to the order in which they are chosen?