Math 180 sample problems for Hour Exam Two

1. Differentiate the following functions:
   (a) $x^2 \ln x$,
   (b) $\sin(a + bx)$,
   (c) $\arctan(3x)$.

2. Differentiate the following functions:
   (a) e^{2-x^2},
   (b) $x \cos(x)$,
   (c) $\arcsin(x/2)$.

3. Let $y = f(x)$ be the function defined implicitly by $y^3 - y + x = 0$ and $f(6) = -2$.
   (a) Find $\dfrac{dy}{dx}$ at the point $(6, -2)$.
   (b) Find the equation of the tangent line at $(6, -2)$.

4. Use the information in the table about f and g to find:
   (a) $h'(0)$, where $h(x) = f(g(x))$.
   (b) $k'(2)$, where $k(x) = f(x)g(x)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	-1	2	5
1	-1	2	4	3
2	7	3	1	4

5. Find the critical points of the function $f(x) = x^3 + 3x^2 - 9x - 11$ and find the global minimum of $f(x)$ on the interval $-4 \le x \le 3$.

6. Find the x- and y-coordinates of all local maxima, local minima, and inflection points of $f(x) = x^3 - 3x + 2$.

7. Find $\displaystyle\lim_{x \to 0} \frac{1 - e^x}{x - x^2}$.

8. You wish to enclose a 400 square-foot rectangular garden with shrubs costing \$40 per foot on three sides and a wall costing \$20 per foot on the fourth side. Find the dimensions that minimize the total cost.

9. Find the largest interval on which $f(x) = (x^2 + 1)e^{-x}$ is concave down.

10. The graph below is of the *derivative* $f'(x)$ on the interval $(-0.5, 5.5)$. Determine the intervals on which the original function f is:
    (a) increasing,
    (b) decreasing,
    (c) concave up,
    (d) concave down.
    (e) Give one value of x at which f has a local maximum.