**Math 180** **Sample Final Exam Questions** **December 2007**

1. Differentiate with respect to $x$. Write your answers showing the use of the appropriate techniques. Do *not* simplify.

$$\text{(a) } x^{1066} + x^{1/2} - x^{-2}, \qquad \text{(b) } e^{\sqrt{x}}, \qquad \text{(c) } \frac{\sin(x)}{5+x^2}.$$

2. Differentiate, writing your answers as in problem 1.

$$\text{(a) } e^{3x}\cos(5x), \qquad \text{(b) } \ln(x^2+x+1), \qquad \text{(c) } \tan\left(\frac{1}{x}\right).$$

3. Differentiate, writing your answers as in problem 1.

$$\text{(a) } x^{2005} + x^{2/3}, \qquad \text{(b) } \cos(\pi x), \qquad \text{(c) } \frac{1+2x}{3+x^2}.$$

4. Differentiate, writing your answers as in problem 1.

$$\text{(a) } x^2 e^{-3x}, \qquad \text{(b) } \arctan(x), \qquad \text{(c) } \ln(\cos(x)).$$

5. Use implicit differentiation to find the slope of the line tangent to the curve

$$x^2 + xy + y^2 = 7$$

at the point $(2, 1)$.

6. Use calculus to find the exact $x$- and $y$-coordinates of any local maxima, local minima, and inflection points of the function $f(x) = x^3 - 12x + 5$.

7. Use calculus to find the $x$- and $y$-coordinates of any local maxima, local minima, and inflection points of the function $f(x) = xe^{-x}$ on the interval $0 \le x < \infty$. The $y$-coordinates may be written in terms of $e$ or as a 4-place decimal.

8. Estimate the integral $\int_0^{40} f(t)\,dt$ using the left Riemann sum with four subdivisions. Some values of the function $f$ are given in the table:

| $t$ | 0 | 10 | 20 | 30 | 40 |
|---|---|---|---|---|---|
| $f(t)$ | 5.3 | 5.1 | 4.6 | 3.7 | 2.3 |

If the function $f$ is known to be decreasing, could the integral be larger than your estimate? Explain why or why not?

9. Write the integral which gives the area of the region between $x = 0$ and $x = 2$, above the $x$-axis, and below the curve $y = 9 - x^2$.
Evaluate your integral exactly to find the area.

10. Write an integral which gives the area of the region between $x = 1$ and $x = 3$, above the $x$-axis, and below the curve $y = x - \frac{1}{x^2}$.
Evaluate your integral to find the area.

11. The average value of the function $f(x)$ on the interval $a \le x \le b$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

Find the average value of the function $f(x) = \frac{1}{x^2}$ on the interval $2 \le x \le 6$.

12. Find

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}.$$

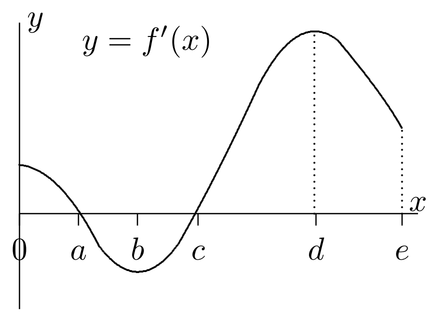
Explain how you obtain your answer.

13. Find

$$\lim_{x\to 0} \frac{1-\cos(3x)}{x^2}.$$

Explain how you obtain your answer.

14. The graph below represents the derivative, $f'(x)$.




(i) On what interval is the original $f$ decreasing?

(ii) At which labeled value of $x$ is the value of $f(x)$ a global minimum?

(iii) At which labeled value of $x$ is the value of $f(x)$ a global maximum?

(iv) At which labeled values of $x$ does $y = f(x)$ have an inflection point?

15. The function $f(x)$ has the following properties:

- $f(5) = 2$,
- $f'(5) = 0.6$,
- $f''(5) = -0.4$.

(a) Find the tangent line to $y = f(x)$ at the point $(5, 2)$.

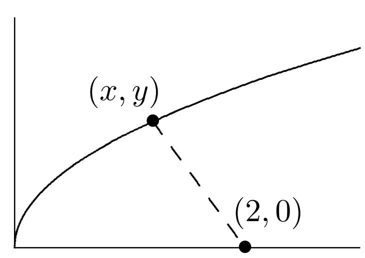
(b) Use (a) to estimate $f(5.2)$.

(c) If $f$ is known to be concave down, could your estimate in (b) be greater than actual $f(5.2)$? Give a reason supporting your answer.

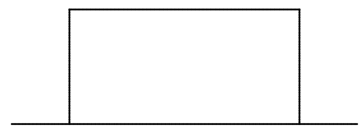
16. The point $(x, y)$ lies on the curve $y = \sqrt{x}$.

(a) Find the distance from $(x, y)$ to $(2, 0)$ as a function $f(x)$ of $x$ alone.

(a) Find the value of $x$ that makes this distance the smallest.



17. You have 24 feet of rabbit-proof fence to build a rectangular garden using one wall of a house as one side of the garden and the fence on the other three sides. What dimensions of the rectangle give the largest possible area for the garden.



18. Find $\int xe^{x^2-1}\,dx$.

19. Find $\int \sin^2 x \cos x\,dx$.

20. Evaluate $\int_2^5 \frac{2x-3}{\sqrt{x^2-3x+6}}\,dx$.