

MATH 180
Fall 2006
2nd Hour Exam
A collection of problems

This is a list of problems to help you prepare for the 2nd hour exam. It is not considered to be exhaustive and you should not expect to find your actual exam problems in the list below. It is to serve as a study aid and it cannot be a substitute for in-class reviews or study of your class notes.

1. Differentiate the functions: (do not simplify)

$$f(x) = x \cos(x^2 + 1) \quad f(x) = xe^{\sqrt{x}} \quad f(x) = \arcsin(3x + 1) \quad f(x) = \frac{e^{3x}}{\ln x}$$

$$f(x) = \cosh(x^2 + 5x + 2) \quad f(x) = \ln(x + \cos x) \quad f(x) = (\ln(\cos x) + 1)^{3/4}$$

$$f(x) = \frac{x \cos x}{\tan x} \quad f(x) = \arctan(3x^2 + 1) \quad f(x) = 2^{x^2+1} - 3^{\ln x}$$

2. A curve is given by $y^3 + 4xy - x^3 + x^2y = 5$.

i) Find the equation of the tangent to the curve at the point $(1, 1)$.

ii) Estimate the y -coordinate of the point that belongs to the curve, has x coordinate 1.03 and lies close to $(1, 1)$.

3. Let $f(x) = \frac{x}{x^2 + 1}$.

- Determine the intervals on which f is increasing and those on which it is decreasing.
- Determine the intervals on which f is concave up and those on which it is concave down.
- Find the critical points of f and determine if they correspond to local extrema.
- Find the asymptotes of f .
- Determine the global extrema of f .
- Sketch the graph of f .

4. Let $f(x) = xe^x$.

i) Find and classify the critical points of f .

ii) Find the global minimum of f over the entire real line.

5. Find the minimum and maximum of the function $f(x) = \sqrt{6x - x^3}$ over the interval $[0, 2]$.

6. Let $f(x) = 3x - x^3$.
- On what interval(s) is f increasing?
 - On what interval(s) is f decreasing?
 - On what interval(s) is f concave up?
 - On what interval(s) is f concave down?
 - Sketch the graph of f
7. For a function $f(x)$ we know that $f(3) = 2$ and that $f'(3) = -3$. Give an estimate for $f(2.91)$.
8. Let $f(x) = \frac{x^2 + 1}{x + 1}$. Find the best linear approximation of f around the point $x = 0$ and use it in order to estimate $f(.2)$. Would this be an underestimate or an overestimate?
9. A rectangular farm of total area 20,000 sq. feet is to be fenced on three sides. Find the dimensions that are going to give the minimum cost.
10. Let $f(x) = 3x^5 - x^3$.
- Find the critical points of f .
 - Determine the intervals on which f is increasing and the ones on which it is decreasing.
 - Determine the intervals on which f is concave up and the ones on which it is concave down.
 - Determine the inflection points of f .
 - Sketch the graph of f .
11. A rectangle has its left lower corner at $(0, 0)$ and its upper right corner on the graph of

$$f(x) = x^2 + \frac{1}{x^2}$$

- Express its area as a function of x .
 - Determine x for which the area is minimum?
 - Can the area of such a rectangle be as large as we please?
12. A box has square base of side x , and constant surface equal to $12m^2$.
- Express its volume as a function of x .
 - Find the maximum volume of such a box.

13. A spaceship is traveling at 300 meters/second (coasting) when it fires its rockets (directly opposite the direction of travel) for a burn of 10 seconds. The burn starts at $t = 0$ seconds and during it, the position of the ship is given by $s(t) = 5t^2 + 300t$ meters from the point of firing.
- (a) Determine the velocity and find $\Delta v = v(10) - v(0)$ (velocity difference before and after burn).
 - (b) Find the acceleration a during the burn.
 - (c) Graph the functions s , v and a on the interval $[-1, 11]$.
 - (d) Find the minimum and maximum values of s , v , and a on that interval.