

Thursday, December 11, 1997

3:30PM–5:30PM

Name (PRINT) \_\_\_\_\_ (1) *Return* this exam copy with your exam booklet. (2) *Write* your solutions in your exam booklet. (3) *Show* your work. (4) There are **ten questions** on this exam. **Check that this copy is complete.** (5) There should be a table of integrals and basic Taylor series on a separate page. **Check that this is the case.** (6) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (20) Find the following indefinite integrals:

$$a) \int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx \qquad b) \int x^3 e^{x^2} dx.$$

*Verify* your answers by differentiation. [Hint: For part b) try integration by parts with  $u = x^2$ .]

2. (20) Find the following indefinite integrals:

$$a) \int \frac{x^2}{x^2 + 1} dx \qquad b) \int \frac{1}{x^2 + x - 6} dx.$$

*Verify* your answers by differentiation.

3. (15) Determine whether or not the following improper integral converges or diverges. If it converges, evaluate it. In any case, *justify* your answer.

$$\int_{-1}^1 \frac{1}{x^2} dx.$$

4. (20) On the moon acceleration due to gravity is  $1.6 \text{ meters/second}^2$ . A rock is dropped from 12 meters above the moon's surface with initial velocity 0. Find:

- the formulas for the velocity  $v(t)$  of the rock and its height  $h(t)$  above the moon's surface at time  $t$  and
- the time  $t$  when the rock hits the surface of the moon.

5. (20) A cylindrical tank 12 feet high with circular base of radius 14 has 7 feet of water. Calculate the amount of work required to pump the water out of the top of the tank. (Recall that water weighs 62.4 pounds/cubic foot.)

**Questions 6–10 on reverse side**

6. (20) Consider the region in the plane bounded by the curve  $y = 4 - x^2$  and the lines  $y = 2 + x$ ,  $x = 0$  and  $x = 1$ .
- Sketch the region.
  - Compute the *volume* of solid obtained by rotating the region about the  $x$ -axis.
7. (20) The density of a rod of length  $L$  is  $e^{-2x}$  per unit length, where  $x$  is the distance from one end. If the mass of the rod is  $1/3$ , find its length  $L$ .
8. (20) One will receive a continuous stream of income starting *now* for the next ten years. For the first five years the income stream will be a constant \$1,000 per year. For the five years after that the income stream will be a constant \$500 per year. If the annual rate of interest is 5% compounded continuously, find the *present* value of the income stream.
9. (25)
- Find the Taylor polynomial of degree 3 approximating  $f(t) = 1/(1+t)$  for  $t$  near 0.
  - Find the Taylor series for  $f(t) = 1/(1 - t^4)$  about  $t = 0$ .
  - Find the first 4 *non-zero* terms of the Taylor series for

$$\int_0^x \frac{1}{1 - t^4} dt$$

about  $x = 0$ .

10. (20) A ball is dropped from a height of 12 feet and bounces straight up and down. After each bounce the ball rises to a height of  $5/8$  of the height of the previous bounce. (After the first bounce the ball rises to a height of  $12(5/8)$  feet and thus after the second bounce the ball rises to a height of  $12(5/8)(5/8)$  feet.)
- Find an expression for *how high* the ball rises after the  $n^{\text{th}}$  bounce.
  - Find a closed form expression for the *total vertical distance* the ball has traveled when it hits the floor for the  $n^{\text{th}}$  time. (Here closed form means a formula which evaluates a sum.)
  - If the ball bounces an *infinite* number of times, what is the total vertical distance the ball travels?