Math 181 HONORS CALCULUS II Final Exam Solution 12/10/98 Radford

1. (20) a) (10 pts) Since $x^2+5x-14=(x-2)(x+7)=(x-2)(x-(-7))$

$$\int \frac{1}{x^2+5x-14}\,dx = \int \frac{1}{(x-2)(x-(-7))}\,dx = \frac{1}{9}\left(\ln|x-2| - \ln|x+7|\right) + C$$

by V.26 of the table (*factorization 4 pts, solve 6 pts*).

b) (10 pts) Let $u = x+5$; thus $du = dx$.

$$\int \frac{1}{x^2+10x+28}\,dx = \int \frac{1}{(x+5)^2+(\sqrt{3})^2}\,dx = \int \frac{1}{u^2+(\sqrt{3})^2}\,du$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right) + C = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+5}{\sqrt{3}}\right) + C$$

by V.24 of the table (*substitution and set up 5 pts; solve 5 pts*).

2. (20) a) (10 pts) Let $u = x^3+5x+6$. Then $du = (3x^2+5)\,dx$. Thus

$$\int \frac{6x^2+10}{x^3+5x+6}\,dx = 2\int \frac{du}{u} = 2\ln|u| + C = 2\ln|x^3+5x+6| + C.$$

(*substitution, set up 5 pts; solve 5 pts*)

b) (10 pts) Let $u = \sqrt{x}$. Then $du = 1/(2\sqrt{x})\,dx$ so $2u\,du = dx$. Therefore

$$\int \cos(\sqrt{x})\,dx = 2\int u\cos(u)\,du = 2(u\sin(u) - \int \sin(u)\,du) = 2(\sqrt{x}\sin(\sqrt{x}) + \cos(\sqrt{x})) + C$$

by III.16 of the table (*substitution and set up 5 pts; solve 5 pts*).

3. (20 pts) $a=-6$ so $v=-6t+v_0$ and thus $s=-3t^2+v_0t+s_0$. Since $v_0=21$ and $s_0=54$ we have $v=-6t+21$ and $s=-3t^2+21t+54$ (*find v, s 10 pts*). a) $v=0$ at this time so $t=7/2$ and thus $s=-3(7/2)^2+21(7/2)+54=(-3{\cdot}49+21{\cdot}7{\cdot}2+54{\cdot}4)/4=363/4=90.75$ feet (*4 pts*). b) $s=0$ so $-3t^2+21t+54=0$, or $t^2-7t+18=0$, or $(t-9)(t+2)=0$. Thus $t=9$ and $v=-6{\cdot}9+21=-33$ feet/second (*time 3 pts; velocity 3 pts*).

4.(20 pts) a) (16 pts) Let x be the distance from the base of the tank to a thin horizontal slab of water Δx in thichness. The volume of the water is $8{\cdot}11{\cdot}\Delta x$ and thus the weight is $62.4{\cdot}(8{\cdot}11{\cdot}\Delta x)$. The slab is lifted $25-x$ feet. Thus the work done is

$$\int_0^9 (62.4){\cdot}8{\cdot}11{\cdot}(25-x)\,dx = (62.4){\cdot}8{\cdot}11{\cdot}(25x - x^2/2)\Big|_0^9 = 1013126.4 \;\; ft{-}lbs$$

(*limits 4 pts; integrand 6 pts; evaluate 6 pts*). b) $\int_0^9 (62.4){\cdot}11{\cdot}x\,dx$ *lbs* (*4 pts*) since the pressure on a strip of thickness Δx at a depth of x feet is $x{\cdot}(62.4){\cdot}11{\cdot}\Delta x$ *lbs*.

5. (20 pts) a) (6 pts) $TRAP(30)=282.3600$ (*3 pts*) and $SIMP(30)=282.0000$ (*3 pts*). b) (6 pts) $f(4)=4^3-3{\cdot}4+2=54$, $f'(4)=3{\cdot}4^2-3=45$ and $f''(4)=6{\cdot}4=24$. Thus

$p_2(x) = 54 + 45(x-4) + (24/2!)(x-4)^2 = 54 + 45(x-4) + 12(x-4)^2$ (*derivatives 2 pts; Taylor polynomial 4 pts*). c) (8 pts) $f'''(4) = 6$ so $f'''(4)/3! = 1$. Note that $f^{(n)}(x) = 0$ for $n \geq 4$. Therefore $\sum_{n=0}^{\infty}(f^{(n)}(4)/n!)(x-4)^n = 54+45(x-4)+12(x-4)^2+1(x-4)^3$ (*higher derivatives 2 pts; Taylor series 6 pts*).

6. (20 pts) (*limits 2 pts; set up 12 pts; evaluate 6 pts*)

$$\int_0^1 \pi(\sqrt{x}+3)^2\,dx - \int_0^1 \pi(x^3+3)^2\,dx = \pi\int_0^1 [(x+6\sqrt{x}+9)-(x^6+6x^3+9)]\,dx$$
$$= \pi(x^2/2 + 6(2/3)x^{3/2} - x^7/7 - (6/4)x^4)|_0^1 = \pi(1/2+4-1/7-3/2)$$
$$= \pi(20/7) \approx 8.97598 \ \textit{units}^3.$$

7. (25 pts) a) (5 pts) We need to solve $1 = \int_0^2 c(2x - x^2 + x^3/9)\,dx$, or $1 = c[(x^2-(1/3)x^3 + (1/36)x^4]|_0^3 = [(9-9+9/4)/3]c = (9/4)c$. Therefore $c = 4/9$. b) (5 pts)

$$P(x) = \begin{cases} 0 & : \ x < 0 \\ (4/9)[x^2-(1/3)x^3+(1/36)x^4] & : \ 0 \leq x \leq 3 \\ 1 & : \ 3 < x \end{cases}$$

c) (7 pts) $1 - P(2) = 1 - (4/9)(4 - 8/3 + 4/9) = 1 - (4/9)(16/9) = 17/81 \approx .20988$ (*formula 3 pts; evaluate 4 pts*). d) (8 pts) $\int_{-\infty}^{\infty} x\rho(x)\,dx = \int_0^3 x(4/9)(2x-x^2+x^3/9)\,dx$ $= (4/9)[(2/3)x^3-(1/4)x^4+(1/45)x^5]|_0^3 = (4/9)(3^3)[2/3-3/4+1/5] = (12)(40-45+12)/60$ $= 7/5 = 1.4$ *years* (*formula 3 pts; evaluate 5 pts*)

8. (20 pts) a) (6 pts) $\int_3^{11} 3500e^{.08(11-t)}\,dt$ (*limits 3 pts; integrand 3 pts*). b) (14 pts)

$$\int_3^{\infty} 3500e^{-.08t}\,dt = 3500(e^{-.08t}/(-.08))|_3^{\infty} = 3500(e^{-.24}/.08) \approx \$34414.96892$$

(*limits 3 pts; integrand 4 pts, evaluate 7 pts*).

9. (15 pts) a) (6 pts) To show $\sum_{n=0}^{\infty} 3n/(n^4+1)$ converges let $f(x) = 3x/(x^4+1)$. Let $u = x^2$; thus $du = 2x\,dx$. Since $\int_1^{\infty} 3x\,dx/(x^4+1) = (3/2)\int_{1^2}^{\infty} du/(u^2+1) = (3/2)\tan^{-1}|_1^{\infty}$ $= (3/2)(\pi/2 - \pi/4)$ the series converges (*function 2 pts; integration 4 pts*). b) (5 pts) $|a_{n+1}/a_n| = (17^{n+1}/(n+1)!)/(17^n/n!) = 17/(n+1) \longrightarrow 0$ as $n \longrightarrow \infty$. Thus $L = 0$ and the series converges. c) (4 pts) $3+9/5+27/25+81/125+\cdots = 3(1+3/5+(3/5)^2+(3/5)^3+\cdots)$ $= 3(1/(1-3/5)) = 15/2 = 7.5$.

10. (20 pts) a) (6 pts) $2(13 + 13(9/11) + 13(9/11)^2 + \cdots + 13(9/11)^{n-1}) = 2{\cdot}13(1-(9/11)^n)/(1-9/11)$ feet. b) (6 pts) $2{\cdot}13(1/(1-9/11)) = 13{\cdot}11 = 143$ feet (*formula 3 pts; evaluation 3 pts*). c) (8 pts) The bounce heights are $13, 13(9/11), 13(9/11)^2, 13(9/11)^3, \ldots$. Therefore the time the ball bounces is (*geometric series 4 pts; evaluation 4 pts*)

$$2(\frac{\sqrt{13}}{4} + \frac{\sqrt{13(9/11)}}{4} + \frac{\sqrt{13(9/11)^2}}{4} + \frac{\sqrt{13(9/11)^3}}{4} + \cdots)$$
$$= \frac{\sqrt{13}}{2}\left(1 + \sqrt{9/11} + (\sqrt{9/11})^2 + (\sqrt{9/11})^3 + \cdots\right)$$
$$= \frac{\sqrt{13}}{2}\left(\frac{1}{1-\sqrt{9/11}}\right) \approx 18.88396 \ \textit{seconds}.$$