Math 181: Calculus II Thurs 9 Dec 1999 **Final Exam** All 8 problems are worth 25 points each.

Remember to SHOW ALL WORK.

1) Evaluate:

a) $\int \frac{x^2}{\sqrt{1+5x^3}}\, dx.$

b) $\int x^{\frac{3}{2}} \ \ln x \ dx.$

2)a) Find a function $F(x)$ such that $F'(x) = x^2 e^{x^3}$ and $F(0) = 2$.

b) Either evaluate $\int_2^\infty \frac{dx}{x(\ln x)^2}$, or show that it diverges.

3) Let R be the finite region bounded by the graphs of $y = x^3$ and $y = 2x^2$. (Thus R is the region between the two points of intersection at $x = 0$ and $x = 2$).

a) Find the area of the region R.

b) Find the volume of the solid obtained by rotating the region R about the x-axis.

4) A tank with rectangular sides and a square base is filled with a thick green liquid. The tank is 25 ft tall, its base is 4 ft by 4 ft, and the liquid weighs 105 lb/ft^3. Find the amount of work done in pumping the liquid to an elevation of 10 ft above the top of the tank.

5) Suppose that the probability density function for a variable x is given by $p(x) = \frac{3}{8}(x-2)^2$ for $0 \leq x \leq 2$ and $p(x) = 0$ elsewhere.

a) Determine the probability that x will be at least 1; that is, find $P(x \geq 1)$.

b) Determine the mean value $\bar{x}$ of the variable x with the given probability distribution $p(x)$.

6) A function $g(x)$ has the Taylor series about $x = 0$ given by

$$2 + \frac{3}{2}x + \frac{4}{2^2}x^2 + \frac{5}{2^3}x^3 + \cdots + \frac{n+2}{2^n}x^n + \cdots.$$

a) Find the fourth derivative of the function $g(x)$ at 0, namely $g^{(4)}(0)$.

b) Find the radius of convergence for this Taylor series.

7)a) Compute the first four nonzero terms of the Taylor series for $\frac{1}{1+x^2}$ about $x = 0$.

b) Using (a) and the fact that $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$, compute the first four nonzero terms of the Taylor series for $\arctan x$ about $x = 0$

8) Let $f(x)$ be the function of period 2π with

$$f(x) = \begin{cases} -1 & \text{if} \quad -\pi < x \leq 0 \\ 1 & \text{if} \quad 0 < x \leq \pi \end{cases}$$

a) Show that the Fourier coefficients a_n (for the cosine-terms) are all 0. (You may give an explanation with or without explicitly evaluating integrals).

b) Find the Fourier coefficients b_n (or at least find b_1, b_2, b_3). c) Using (a) and (b), find the third Fourier approximation of $f(x)$.