Math 181 Spring 2000 Final Exam
(This is a long exam, but has many good problems for exercise!)

1. (10 pts). Find the following derivative: $\dfrac{d}{dx}\displaystyle\int_1^{2x} \arctan(t)\,dt$.

2. (8 pts. each) Evaluate the following integrals. Show your work, noting what substitutions you make, what parts you use, what formulas from the table you use, etc.

   (a) $\displaystyle\int_0^{12} x\sqrt{25+x^2}\,dx$

   (b) $\displaystyle\int (\ln(t))^2\,dt$

   (c) $\displaystyle\int e^{2x}\cos(3x)\,dx$

   (d) $\displaystyle\int \frac{1}{y^2-5y+6}\,dy$

   (e) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$

3. (a) (15 pts. — 5 pts. each) Write down at least four nonzero terms of the Taylor series about $x=0$ for the following functions:

   $$1.\ \sin(x) \qquad 2.\ \frac{1}{1-x^2} \qquad 3.\ \ln(1+x).$$

   (b) (10 pts.) Write down the first four nonzero terms of the Taylor series for the function $e^{2x}\cos(x)$.

4. (25 pts.) Show that the volume of a sphere of radius 1 is $\dfrac{4\pi}{3}$.

5. (25 pts.) A rectangular water tank has length 20 feet, width 10 feet and depth 15 feet. Water weighs 62.4 lb/ft^3. If the tank is full, how much work is required to pump all of the water to the top of the tank?

6. (25 pts.) Find the present value of an income stream of \$20,000 per year for twenty years assuming an interest rate of 7% a year compounded continuously for the first 10 years and an interest rate of 8% a year compounded continuously for the last 10 years.

7. (25 pts.) Suppose that $f(0) = 3$, $f'(0) = -1$, $f''(0) = -2$, $f^{(3)}(0) = 0$, and $f^{(4)}(0) = 2$.

   (a) Write down the Taylor polynomial of degree four for f about $x = 0$.

   (b) Use (a) to approximate $f(0.1)$

8. (25 pts.) Find the Fourier coefficients a_0, a_1, a_2, a_3 and b_1, b_2, b_3 for the function $f(x)$ which is periodic of period 2π and which is given on $[-\pi, \pi]$ by

$$f(x) = \begin{cases} 0 & \text{for } -\pi \leq x < -\frac{\pi}{2}, \\ 2 & \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, \\ 0 & \text{for } \frac{\pi}{2} < x \leq \pi. \end{cases}$$

9. (10 pts. Extra Credit) Find all the Fourier coefficients for the function in the previous problem.