

Math 181 - FINAL EXAM

Name: _____

Spring 2002

SS#: _____

T.A.: _____

(1) (40 pts.) Find the following indefinite integrals.

(a) $\int \frac{x}{x^2 + x - 6} dx$

(b) $\int x\sqrt{x^2 + 36} dx$

(c) $\int (\ln x)^2 dx$

(d) $\int \frac{\cos(\sqrt{x+1})}{\sqrt{x+1}} dx$

(2) (20 pts.) Find the volume of the solid generated by rotating the region bounded by $y = 4x$ and $y = x^2$ about the line $y = -1$.

(3) (20 pts.) Let $f(x)$ be a periodic function of period 2π . In $[-\pi, \pi]$, $f(x)$ is given by

$$f(x) = \begin{cases} 0 & -\pi < x < -\pi/2 \\ \pi & \text{for } -\pi/2 \leq x \leq \pi/2 \\ 0 & \pi/2 < x \leq \pi. \end{cases}$$

(a) State the integral formula computing the Fourier coefficients a_i 's and b_i 's for the function $f(x)$.

(b) Find the Fourier coefficients a_i for $0 \leq i \leq 4$ and b_i for $1 \leq i \leq 4$.

(4) (20 pts.) Let $f(x) = \sum_{n=0}^{\infty} \frac{n}{3^n} x^n$

(a) Find $f^{(4)}(0)$.

(b) Find the radius of convergence for the Taylor series of $f(x)$.

(5) (30 pts.) A group of patients has received a treatment of cancer. Let t be the survival time, the number of years a patient lives after receiving the treatment. The density function for the distribution of t is $p(t) = 0.2 e^{-0.2t}$.

(a) Find the probability that a patient will survive more than 4 years.

(b) Find the mean of the distribution function $p(t)$.

(c) Find the median of the distribution function $p(t)$.

- (6) (20 pts.) A 400 lb. weight is being lifted to a height of 20 feet off the ground by workers standing on the roof of a building. The roof is 60 feet above the ground, and the rope that the workers use weighs 4 lbs. per foot. Find the work done by the workers to lift the weight.
- (7) (30 pts.) (a) Find the Taylor polynomial $P_2(x)$ of degree 2 for e^{-x} about $x = 0$.
- (b) Use $P_2(0.1)$ to estimate $e^{-0.1}$. Estimate the error $|e^{-0.1} - P_2(0.1)|$.
- (c) Estimate the error $|e^{-x} - P_2(x)|$ for x in $[0, 0.1]$.
- (8) (20 pts) Consider the infinite series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$. Use the integral test to determine whether the series converges.