

## Calculus III Midterm I

February 16, 2001

1. Given two vectors  $\vec{\mathbf{a}} = \langle -3, 2, 2 \rangle$ ,  $\vec{\mathbf{b}} = \langle 4, 3, -1 \rangle$ .

a) Find a unit vector in the same direction as  $\vec{\mathbf{a}}$ .

b) Find the angle between  $\vec{\mathbf{a}}$  and  $\vec{\mathbf{b}}$ .

2. Find the equation of the plane determined by the three points  $(0, 0, 0)$ ,  $(1, 0, 0)$ , and  $(2, 3, 4)$ .

3. The position vector of a moving particle is given by

$$\vec{\mathbf{r}}(t) = \langle 3t - 4, 3t^2, 2t^2 + t \rangle.$$

(a) Find the velocity  $\vec{\mathbf{v}}(t)$ .

(b) Find the speed.

(c) Find the acceleration  $\vec{\mathbf{a}}(t)$ .

(d) Find the curvature  $\kappa(t)$ .

(e) Write the integral which gives the arclength from the point where  $t = 0$  to the point where  $t = 5$ , do not evaluate the integral.

4. Let

$$f(x, y) = ye^{(x^2+y^2)}.$$

a) Find  $f_x$  and  $f_y$ .

b) Find  $f_{x,y}$ .

5. Let

$$f(x, y) = \frac{2x}{x - y}.$$

a) Find the domain of  $f$ .

b) Sketch the level curves  $f(x, y) = k$  for  $k = 0, 1, 2$  and label them.

6. a) Find an equation of the tangent plane to the surface

$$z = x + \ln(2x + y)$$

at the point  $(-1, 3, -1)$ .

b) Find the differential of the function  $f(x, y) = x + \ln(2x + y)$ .

**HAND IN THIS SHEET WITH YOUR EXAM**