Math 210, Calculus III **Sample Second Hour Exam**

1. Let $f(x, y) = \sqrt{xy}$.

   (a) (10 points) Find the equation of the tangent plane to the graph of f at the point where $x = y = 1$.

   (b) (10 points) Use your result from part (a) to estimate $\sqrt{1.32}$. (HINT: $1.32 = 1.1 \times 1.2$)

2. (15 points) Find and classify the critical points of the function

$$f(x, y) = x^3 - 3xy + y^3.$$

3. Let $f(x, y, z) = ze^{x-y}$

   (a) (5 points) Find a unit vector $\vec{u}$ that points in the direction from $(0, 1, 2)$ toward $(1, 0, 3)$.

   (b) (10 points) Find the directional derivative $D_{\vec{u}}f$ of f at the point $(0, 1, 2)$ in the direction of $\vec{u}$.

4. (20 points) Use Lagrange multipliers to find the rectangle of maximum perimeter which has sides parallel to the coordinate axes and can be inscribed in the ellipse $3x^2 + y^2 = 1$.

5. (15 points) Use a double integral to calculate the area of the bounded region determined by the curves $y = x$ and $y = x^2 - 4x + 4$.

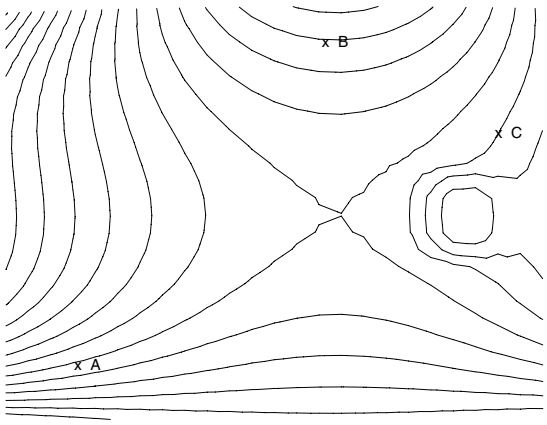
PROBLEM 6 IS ON THE BACK!

6. The picture below shows a contour map of a function $g(x, y)$. The function g has exactly two critical points in the region shown. One is a saddle and the other is a local maximum.

   (a) (5 points) At each of the points A, B, and C, draw an arrow on the picture in the direction of the gradient vector ∇f.

   (b) (5 points) At which of the points A,B, or C is the magnitude of ∇f largest?

   (c) (5 points) Draw the path of steepest ascent starting from the point B.



DON'T FORGET TO HAND THIS SHEET IN WITH YOUR EXAM!