Sample Third Hour Exam

1. (15 pts) Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane $x/2 + y + z/2 = 1$.

2. (10 pts) Rewrite the integral

$$\int_0^2 \int_{x^4}^{4x^2} f(x,y)dydx$$

as an iterated integral in the order $dxdy$.

3. (15 pts) The semicircular lamina bounded by the x-axis and the upper half of the circle $x^2 + y^2 = 4$ has a density of $\rho(x,y) = y$. Find the total mass of the lamina.

4. (15 pts) Use spherical coordinates to evaluate the integral:

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)^2 dzdydx.$$

5. (15 pts) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F}(x,y) = \langle -xy, x^2 \rangle$ and C is the part of the parabola $y = x^2$ that goes from (1,1) to (2,4).

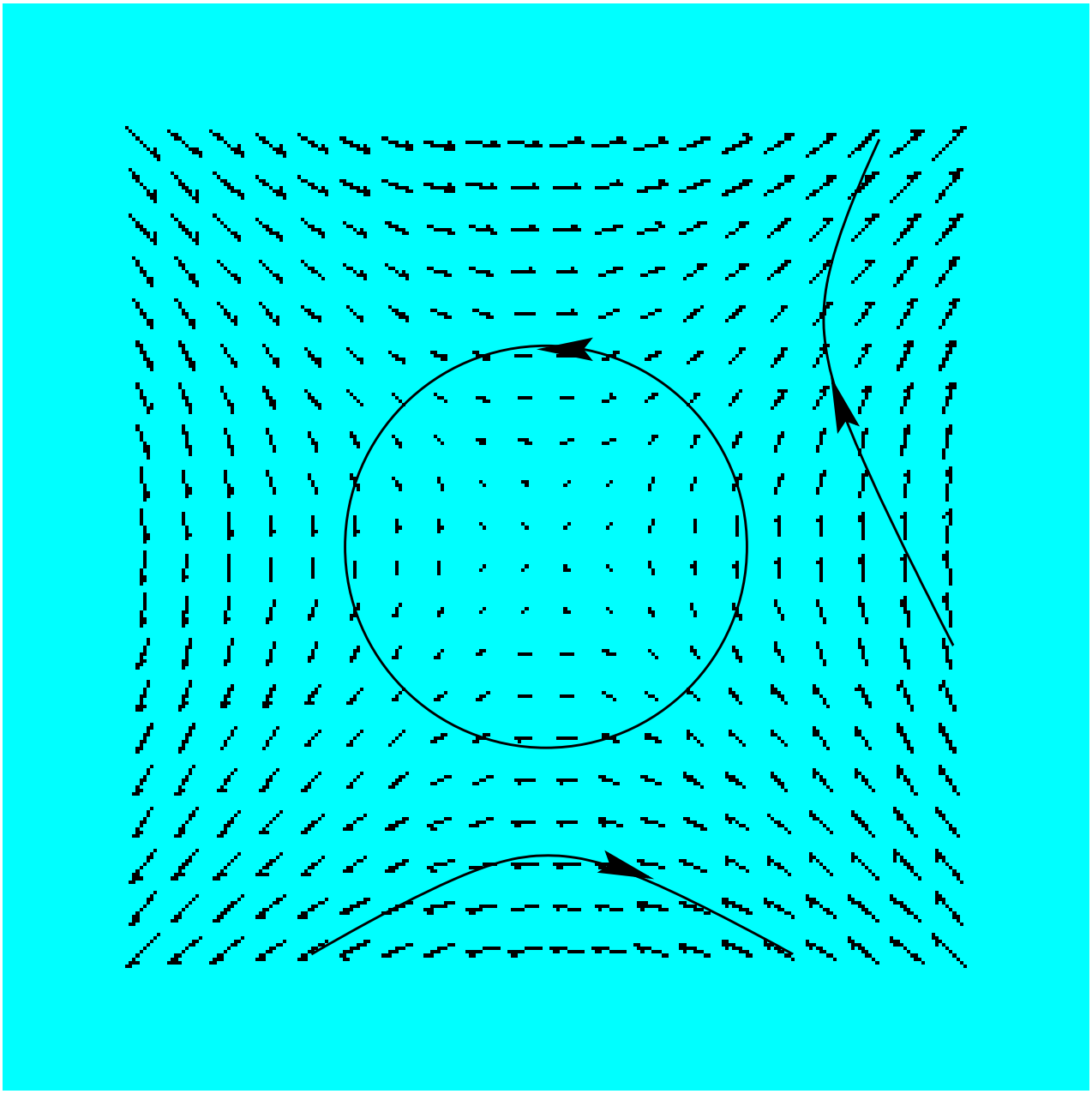
6. (15 pts) Show that one of the following vector fields is the gradient of some function h(x,y) (i.e. is conservative) and the other can not be a gradient.

$\mathbf{F}(x,y) = y^2\,\mathbf{i} + xy\,\mathbf{j}$

$\mathbf{G}(x,y) = (x^2y^3 - y)\,\mathbf{i} - (x - x^3y^2)\mathbf{j}$

PROBLEM 7 IS ON THE BACK

7. (15 pts) For each of the three curves drawn on the vector field $\mathbf{F}(x, y)$ below, indicate whether $\int_C \mathbf{F} \cdot d\mathbf{r}$ will be positive, negative or zero.



DO NOT FORGET TO HAND IN THIS SHEET WITH YOUR EXAM!