

# Final Exam

Math 210H

December 9, 1993

**Problem 1:** (20 points) For the space curve given by  $\vec{c}(t) = \langle e^t, \sin t, \cos t \rangle$ .

- Find the velocity vector  $\vec{v}(t)$  and acceleration vector  $\vec{a}(t)$ .
- Find a unit vector  $\vec{u}$  perpendicular to the velocity and acceleration vectors at  $t = 0$ .

**Problem 2:** (20 points) Let

$$F(x, y, z) = \left( \begin{array}{c} x \cdot y \cdot z \\ x^2 + y^2 + z^2 \end{array} \right)$$

- Calculate the derivative matrix  $DF$  at  $(x_0, y_0, z_0) = (1, 2, 3)$
- Use the first order Taylor approximation to  $F(x, y, z)$  at  $(1, 2, 3)$  to find the approximate value for  $F(1.1, 1.9, 2.8)$ .

**Problem 3:** Find the equation of the tangent plane at  $(1, 2, -1)$  to the level surface

$$x^2 + xyz + 2z^3 + 3 = 0$$

**Problem 4:** a) Compute the directional derivative at  $(x_0, y_0, z_0) = (2, 3, 1)$  of

$$f(x, y, z) = \frac{1}{x^2 + 2y^2 + 3z^2}$$

- What direction from  $(2, 3, 1)$  should we go in order to make  $f(x, y, z)$  decrease fastest?

**Problem 5:** Use the Lagrangian multiplier method to find the maximum and minimum value of the function

$$f(x, y) = 2x^2 + 3y^2 + x$$

constrained to the unit circle  $x^2 + y^2 = 1$

**Problem 6:** (20 points) Find all of the critical points of the function

$$f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$$

and use the second derivative test to classify them as local maxima, local minima, or saddles, or state that the second derivative test fails.

**Problem 7:** Compute the *divergence*  $\nabla \cdot F$  and the *curl*  $\nabla \times F$  of the vector field

$$\vec{F} = \langle 2xye^z, e^z x^2, x^2 ye^z + z^2 \rangle$$

**Problem 8:** Find the arclength of the curve

$$\vec{c}(t) = \langle t, t \sin t, t \cos t \rangle, \quad 0 \leq t \leq \pi$$

**Problem 9:** Change the order of integration, and then calculate

$$\int_0^1 \int_y^1 e^{-x^2} dx dy$$

**Problem 10:** Calculate the total mass of the region bounded by the graphs of  $z = x^2 + y^2 - 8$  and  $z = 10 - x^2 - y^2$  with density  $\rho(x, y, z) = x^2 + y^2$ .

**Problem 11:** Calculate the line integral

$$\int_C y dx + (x + z) dy + y dz$$

along the curve  $\vec{c}(t) = \langle t, t^2, t^3 \rangle$  for  $0 \leq t \leq 1$ .

**Problem 12:** Use Green's Theorem to evaluate the line integral of the plane vector field

$$\vec{F}(x, y) = \langle 2y + e^x, x + \sin(y^2) \rangle$$

along the curve  $\vec{c}(t) = \langle 3 \cos 2t, 3 \sin 2t \rangle$  for  $0 \leq t \leq \pi$ .