Math 210 **Final exam** December 6, 2001

Problem 1:(25 points) For the curve $\vec{r}(t) = \langle -6\cos t, 6\sin t, 8t \rangle$

a) Find the velocity $\vec{v}(t)$ and the acceleration $\vec{a}(t)$.

b) Find the curvature $\kappa(t)$.

Problem 2: (25 points) Let $f(x, y) = y\sin(xy^2)$

a) Compute the directional derivative of f at $(0, 1)$ in the direction of $(4, 3)$.

b) In what direction does f increase most rapidly at the point $(0, 1)$?

Problem 3: (25 points) Use the method of Lagrange multipliers to find the maximum and minimum value of the function

$$f(x, y, z) = x + 3y - z$$

subject to the constraint $x^2 + 4y^2 + z^2 = 17$.

Problem 4: (25 points) Use the transformation: $x = \frac{u+v}{2}$, $y = \frac{v-u}{2}$ to evaluate the integral

$$\int\int_R (x - y)(x + y)dxdy$$

where R is the region bounded by the lines $y = x + 2$, $y = x$, $y = -x + 2$ and $y = -x + 4$.

Problem 5: (25 points) Consider the vector field $\mathbf{F}(x, y) = \langle 1 + 2ye^{2x}, 2y + e^{2x} \rangle$

a) Show that $\mathbf{F}$ is conservative.

b) Find a function f such that $\mathbf{F} = \nabla f$.

c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C given by $\mathbf{r}(t) = \langle 1 - t, te^t \rangle$ for $0 \leq t \leq 1$.

Problem 6: (20 points) Use Green's Theorem to evaluate

$$\oint_C x^2 dx + (2x + y)dy$$

where C is the circle $x^2 + y^2 = 9$.

Problem 7: (20 points) Evaluate

$$\int\int\int_R \frac{dV}{x^2+y^2+z^2}$$

where R is the region between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$.

Problem 8: (20 points) Find an equation of the plane passing through the points $(3,2,1)$, $(2,1,-1)$ and $(-1,3,2)$.

Problem 9: (15 points) Change the order of integration:

$$\int_0^2 \int_{x^2}^{2x} f(x,y)dydx$$