Calculus III Final Exam

May 3, 2001

1. The position function of a particle moving through $\mathbf{R}^3$ is $\vec{r}(t) = \langle a\cos t, b\sin t, t\rangle$.
   a) Compute the velocity, speed, and acceleration of the particle.
   b) What is the length of the arc traversed by the particle from $t = 0$ to $t = 1$?

2. Let $\vec{u} = \langle 0, 0, 1\rangle$ and $\vec{v} = \langle 1, 1, 1\rangle$.
   a) Find the equation of a plane which contains the point $(1, -1, 2)$ and the vectors $\vec{u}$ and $\vec{v}$.
   b) Suppose l is a line in the direction of the vector $\vec{v}$ which contains the point $(1, -1, 2)$. Find equations for the line l.

3. Let $f(x, y, z) = (z^2 + z\sin(x - 3y))^2$. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$.

4. Find the directional derivative of the function $f(x, y, z) = x^2y - 2xz + yz^3$ at the point $(1, -2, 1)$ in the direction $\mathbf{i} + \mathbf{j} - \mathbf{k}$.

5. Let $f(x, y) = x^2 - y^2 + 2y$
   (a) Find the critical points and classify them as relative max, min, or saddle points.
   (b) Find the maximum and minimum of f under the constaint $x^2 + y^2 = 1$.

6. Calculate the iterated integral by first reversing the order of integration.

$$\int_0^1 \int_{x^2}^1 (x\sin y^2)\, dy\, dx$$

7. Find the area of the portion of the plane $2x - 2y + z = 1$ inside the cylinder $x^2 + y^2 = 4$.

8. Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ for the field $\vec{F} = \langle yz, xz, xy\rangle$ and the curve parameterized by $\vec{r}(t) = \langle t, 2t, t^2\rangle$ for $-1 \leq t \leq 1$.

9. a) Show that $\vec{G}(x, y) = \langle xe^{xy}, ye^{xy}\rangle$ is not a conservative vector field.
   b) Show that $\vec{F}(x, y) = \langle ye^{xy}, xe^{xy}\rangle$ is conservative and find a function f such that $\vec{F} = \nabla f$.
   c) Let C be the arc of the ellipse $x^2 + 4y^2 = 16$ from $(0, 2)$ to $(-4, 0)$. Evaluate $\int_C \vec{F} \cdot dr$ where $\vec{F} = \langle ye^{xy}, xe^{xy}\rangle$.

10. Evaluate $\oint_C xy\, dx + x^2e^y\, dy$ where C is the square with verticies $(3, 0)$, (3,2),$(1, 2)$, $(1, 0)$.