



January Special Session in New Orleans

The themes in this year's special session on Mathematics and Education Reform at the Joint Mathematics Meetings in New Orleans are introduced here. The four co-organizers, William Barker, Bowdoin College, Jerry Bona, University of Texas at Austin, Naomi Fisher, University of Illinois at Chicago, and Kenneth Millett, UC Santa Barbara, were joined by Al Cuoco, EDC, Samuel Rankin, AMS, and Eric Robinson, Ithaca College, in co-organizing sections of the program.

Mathematics Immersion Experiences in Professional Teacher Development Programs,
co-organized by Al Cuoco, EDC

An important part of the mathematics one teaches is an understanding of what it is to *do* mathematics. One of the best ways to convey this understanding to pre-college teachers is through an "immersion experience" in mathematics which allows them to investigate one area of mathematics in depth. Using theoretical analyses and concrete examples, this section addresses *Why should such an immersion experience improve one's day-to-day teaching practice?*

Professional Master's Degrees in Mathematics Departments,
co-organized by Samuel Rankin, AMS

Professional Master's Degrees in mathematics, stand-alone master's degrees to prepare students for a specific career path, are options for graduate study for mathematically able students who may not be interested in traditional academic careers. Together the five presentations in this section survey Professional Master's programs in mathematics departments, and consider ways in which departments may implement their own programs.

The Curriculum Foundations Project and Calculus: Evaluating Instructional Effectiveness

What input about the first two years of collegiate mathematics might partner disciplines from the sciences and quantitative social science give mathematics faculty? CRAFTY (Calculus Reform And the First Two Years) has been investigating this question in a series of twelve disciplinary workshops. Discussions from four of the workshops are featured:

statistics, technical mathematics, interdisciplinary programs, and mathematics education.

In addition, two talks address some of the concerns about the teaching of Calculus. One talk looks at what has been found out about student learning and instructional changes in the decade 1988-98; the other discusses an international study of Calculus students in the U.S. and Japan.

Increasing the Number and Diversity of Teachers of Mathematics

The need for teachers from underrepresented groups and for those with the skill and desire to successfully teach in middle schools and high schools with limited resources or serving large minority or disadvantaged communities is especially critical. The four projects presented in this section each address these issues in their own distinct settings. In presenting their work, the speakers discuss their successes, and the obstacles they face in trying to respond to the increasing shortage of teachers.

Secondary Standards-based Curricula: Content and Implementation,

co-organized by Eric Robinson, Ithaca College

The changes in secondary mathematics education that have been under development in many school districts across the country directly effect the mathematical knowledge of incoming college students and the mathematical capabilities of these students. This section starts a dialogue on secondary mathematics by inviting mathematicians to consider (a) the goals of the five curricular projects funded by the NSF, (b) how each of two projects have attempted to meet the goals, and (c) implementation issues in making changes in secondary mathematics. ■

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from the editor

The two featured articles in this issue, *Mathematical Knowledge for Teaching* by Brad Findell (opposite page) and *Focusing on Secondary Mathematics* by Eric Robinson (page 4) bring to mind the original focus of MER to involve mathematicians in the improvement of *precollege* education. Findell's essay poses the problem of how teachers' mathematical knowledge can be brought to bear on students' learning mathematics. (Although he doesn't identify explicitly the school level being addressed, elementary school is certainly one focus.) The paradigm of a class structure in which "teachers help students make connections between their previous knowledge and important mathematical content[,...]taking advantage of [students'] productive informal understandings and encouraging modification of understandings that are mistaken or unproductive" requires more than outstanding pedagogical ability and content knowledge of mathematics. It requires that one's mathematical knowledge is flexible and that concepts are networked complexly. And it requires excellent listening ability and improvisational skills to respond meaningfully to student proffered solutions. How can mathematicians contribute to nurturing these abilities in elementary school teachers? I think a carefully considered response entails reflecting on how to configure multiple conceptions for a problem, and our own classroom environments.

Robinson underscores the inter-related nature of teaching mathematics at the K-12 levels and the college level in his overview of the NSF funded high school curricula. The article is conceived as an introduction and invitation to mathematicians to study the content

of these curricula and discuss—across the secondary college divide—the implications for teaching high school graduates of these programs at the college level. Clearly, the discussion should be interactive so that each group can recognize their inter-dependence with one another. Both sides need to be heard—and listened to.

Mathematicians' involvement in precollege education is a moving target. Precollege mathematics continues to change in response to pressures from within and outside the K-12 system. In turn, these changes have an impact on higher education, and so the cycle continues.

Naomi D. Fisher

The **Mathematicians and Education Reform (MER) Forum** seeks the effective participation of mathematicians in mathematics education reform at the K-12, undergraduate, and graduate levels, and the recognition of the importance of these efforts to the well being of the mathematics community. The MER Forum envisages the pursuit of educational reform through informed discussion of educational issues, thoughtful responses to changing educational conditions, and the promotion of exemplary programs. The creation and support of a network of mathematicians with a sustained commitment to mathematics education is central to this vision.

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Notes on Mathematical and Teaching Ideas

Mathematical Knowledge for Teaching

by Brad Findell, Mathematical Sciences Education Board, National Research Council

What mathematical knowledge is required for teaching? And how might teachers come to know it? Clearly teachers must know the mathematics they teach. But what more mathematics do they need? These questions have occupied mathematicians, mathematics educators, and mathematics education researchers for decades. Most of the work has been based on expert opinion or consensus. These efforts typically have resulted in lists of courses to be included in a prospective teacher's undergraduate program. But the outcome has been disappointing. In particular, research has failed to demonstrate that this sort of preparation leads to better student achievement. It seems that either the teachers fail to learn the content of these courses, or they do learn it but find that it doesn't connect in any recognizable way with their classroom practice. This essay discusses recent approaches that shed some new light on these questions, building primarily on the work of the Mathematical Sciences Education Board at the National Research Council.

Current conceptions of teaching emphasize that effective learning environments are both learner centered and knowledge centered (Bransford, Brown, and Cocking, 1999). Skilled mathematics teachers help students make connections between their previous knowledge and important mathematical content. Effective teaching builds upon what students know, taking advantage of productive informal understandings and encouraging modification of understandings that are mistaken or unproductive.

What knowledge does it take to teach in this way? Predominant conceptions of teacher preparation often separate knowledge for teaching into two categories: pedagogical knowledge and content knowledge. But effective teachers make connections between particular students and particular content. Furthermore, making such connections requires specialized knowledge that lies in the gap between content and pedagogy. Shulman tries to bridge this gap with the term *pedagogical content knowledge*—"an understanding of how particular topics, problems, or issues are organized, presented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (1987, p. 8). If pedagogical content knowledge is a useful construct, then the problems of teacher preparation are two-fold. First, pedagogical content knowledge is unarticulated and implicit in

teaching and teacher education. Second, because content and pedagogy are delivered by different departments in most universities, there is no obvious place where teachers might acquire pedagogical content knowledge.

Within the last decade, a handful of mathematics education researchers, mathematicians, teacher educators, and professional developers have begun to seek solutions to this problem through work based on the principle that teacher education must be situated in classroom practice. This idea was investigated at a workshop hosted by the Mathematical Sciences Education Board in March 1999 (see National Research Council, in press). Ferrini-Mundy and Findell describe the approach as follows:

Analyze the practice of teaching and determine what mathematical knowledge teachers draw upon in their practice. Then use real mathematical "problems" of mathematics teaching practice as sites for learning mathematics, taking advantage of the mathematical opportunities that emerge while working on the problems....

The next step would be to design a curriculum around such activities, in much the same way one might create a mathematics curriculum for engineers or social scientists by looking at the mathematical problems they have to solve. (Ferrini-Mundy & Findell, in press)

They call this an *emergent approach* in the sense that the mathematical content emerges from the work of teaching. For both prospective and practicing teachers, examples from teaching practice are provided through the use of student curriculum materials, samples of students' written work, or through videotapes or written case studies of classroom episodes. An extended example is included below to show that there are mathematical problems to solve in the context of teaching and to make the case that there are important mathematical ideas that lie in the gap between pedagogy and traditional mathematics content.

The Orange Juice Problem

The following example is taken from a 7th grade unit on rational numbers and proportional reasoning (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1997, pp. 27-28). Students are asked to compare four orange juice recipes to

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Focusing on Secondary Mathematics

by Eric Robinson, Ithaca College

Setting the Stage

It was 1989 when *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*¹ urged us to revitalize mathematics education in this country. There was substantial evidence that, by in large, we were not doing an adequate job at any level. But this report did not simply suggest that we needed to do a better job at what we were doing. Backed by such reports as the 1983 document *A Nation at Risk*², it stressed that we had “a mathematics curriculum conforming to the past, blind to the future, and bound by a tradition of minimum expectations.”³ The curriculum had to change to meet the needs of students as individuals and our nation as a whole in a new century. While *Everybody Counts* built on some nascent and existing reform efforts, as well as anticipating and suggesting others, it presented a unique view of mathematics education, kindergarten through graduate school, as one system.

Thinking about mathematics education as a single system brings to mind the interrelationships between the various components of the system. Moreover, recommendations in *Everybody Counts* and in a raft of other documents suggest that improvement in mathematics education requires consideration of *all* components of the educational process. Inside the classroom, these components include, but are not limited to: the curriculum—what students should know and be able to do, how students learn, the preparedness of teachers, and how to measure student and programmatic success. Outside the classroom, these components include issues pertinent to various stakeholders at each educational level. Faculty in colleges and universities who teach mathematics courses are among these stakeholders.

It may seem that the ideal procedure would be to have a single committee or group, with a knowledgeable and comprehensive overview of the education system, build consensus among all the players (inside and outside the classroom) about what changes are needed. Then implement these changes all at once, since they would interrelate with one another. Zip out the old and zip in the new. But there are counter-forces at work when trying to build consensus within and across K-12 education, a decentralized national educational system steeped in a strong tradition of state or more local autonomy, and higher education, which is based on institutional autonomy.

Systemic effort to improve mathematics education “must proceed steadily for many years, on many levels simultaneously...”⁴ That is what has occurred and is still occurring at all levels of the educational continuum. The issues regarding such changes are many and complex. As with many large tasks, a practical method for tackling the task is to have subgroups work on parts of the whole. This is more than a scheduling or parallel-processing problem of huge proportions when we are talking about systemic change in education. Since many of changes require years of development, unless we are willing to wait decades, we cannot always wait for one group to implement a set of recommendations or solutions before other groups affected by the recommendations begin their work. Thus, we move forward in less than ideal ways. But we progress.

It is crucial that independent groups working in different areas of the system share their progress, not only among the various groups, but also with general audiences of stakeholders in an attempt to create deep under-

standing of initiatives and the affects of the changes being made. The MER sessions at the Joint Meetings, and other events like them, can help to establish or enhance a vital communication link between secondary and undergraduate mathematics education that is particularly weak due, in part, to the nature of our educational system. One could make the case that school teachers’ perceptions of what colleges and universities want in terms of student capabilities is often simply based on looking at types of questions on high stakes assessments (e.g. SAT, ACT, AP, and possibly Math Placement exams). One could further argue that the understanding of undergraduate mathematics faculty (who do not interact with colleagues involved in mathematics education at the secondary level) with regard to student capabilities comes from simply looking at test scores or course placements based on such tests. If we want to communicate what secondary students know and are able to do as a result of changes in the secondary curriculum, these high stakes tests tell little of the picture. College and university faculty need more and better information if they want to take advantage of the wide range of skills and abilities that secondary students bring to the undergraduate classroom after successfully completing a more modern curriculum.

Design of Newer Secondary Mathematics Programs: An Overview

In 1989, efforts to reform the teaching and learning of calculus at the undergraduate level were underway. That year also saw the publication of the *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of

Mathematics (NCTM), the largest professional society for teachers of mathematics. That document was the first of three such documents from NCTM, which set forth a new vision of school mathematics for the ensuing decade. *Professional Standards for Teaching Mathematics* and *Assessment Standards for School Mathematics* were published by NCTM in 1991 and 1995 respectively. In addition to setting goals, these documents were intended to promote change with regard to what is taught, how it is taught, how students are assessed and how programs are evaluated. Even though curriculum development could be based on a list of other standards, current use of the term “standards-based curriculum” usually refers to curricula that reflect the goals set forth in NCTM’s *Curriculum and Evaluation Standards* or in the updated NCTM document *Principles and Standards for School Mathematics* (PSSM). This latter document, released last April, is a combined revision of its three predecessors. While there are some changes, the fundamental messages in the PSSM remain the same as they were in the preceding documents.

Many of the statements in the *Curriculum and Evaluation Standards* and in the new PSSM harmonize with sentiments expressed in *Everybody Counts* and other documents that suggested change in the undergraduate curriculum. So, it should not be a surprise that some of the tenets for change in some undergraduate courses have their counterpart in the reshaping of K-12 mathematics education. Here are two obvious parallels. Undergraduate mathematics educators heard that calculus courses should be “a pump, not a filter.” In the NCTM *Standards* documents, the phrase is that “*all students*” need a high quality mathematics education. We also heard that calculus should become “lean and lively.” In the analysis (1995) of the Third International Mathematics and Science Study (TIMSS), we heard that one of the reasons our nation’s school

mathematics students fared poorly when compared to students of 41 other nations was that our traditional mathematics curriculum was “a mile wide and an inch deep.”

In the early 1990’s the National Science Foundation (NSF) requested proposals for new instructional materials aimed at improving school mathematics at each of the elementary, middle grades and high school levels. The Request for Proposals (RFP) for the elementary level came first, followed by an RFP for middle school and later high school. As part of the criteria for consideration, proposals had to reflect attention to: up-to-date content, the latest knowledge concerning how students learn and effective methods of teaching, the appropriate incorporation of technology as both a teaching and learning tool, a focus on depth of understanding rather than breadth, and existing curricular frameworks and standards. With respect to this latter criterion, all proposals that finally received funding considered the existing NCTM *Standards* documents (at least the first two NCTM publications). The awards included funding for five multi-year secondary mathematics proposals intended to replace the algebra I-geometry-algebra II-precalculus sequences that were prevalent in our high schools. Because of the comprehensive curricular nature of these particular programs, they serve as significant examples of alternatives to what was, and in many cases still is, the status quo.

Each of the five programs was in development for a period of at least five years. The design and review teams in each case consisted of practicing teachers, professional mathematicians, teacher educators, educational researchers, evaluators, and, in some cases, other professionals including school administrators, assessment specialists, and business people. The development process for the instructional materials included several phases: an initial design phase

followed by a pilot test phase, a review and redesign phase, a larger field test and evaluation phase, and a final review and redesign phase *before* the instructional materials reached the hands of commercial publishers.

The following list indicates each program by its NSF project name, the name of the major NSF grantee institution, the commercial name and publisher, and the number of school years the program is designed to encompass:

- *ARISE* out of COMAP, published as *Mathematics: Modeling Our World* by W.H. Freeman (4 years)
- Core-Plus Mathematics Project (CPMP) out of Western Michigan State University, published as *Contemporary Mathematics in Context* by Everyday Learning-McGraw Hill (4 years)
- Interactive Mathematics Project (IMP) out of San Francisco State University, published as the *Interactive Mathematics Program* by Key Curriculum Press (4 years)
- MATH Connections developed at CBIA, published as *MATH Connections: A Secondary Mathematics Core Curriculum* by It’s About Time (3 years)
- SIMMS out of Montana State University, published as *Integrated Mathematics: A Modeling Approach Using Technology* by Pearson Custom Publishing (4 years).

These programs offer five different curriculum models and somewhat distinct answers to the question: *what should a student know and be able to do as a result of successfully completing a secondary school mathematics education?* Each program reflects significant changes in every aspect of the classroom: the mathematical content itself, the way that content is encountered and the pedagogy, the use of technology, and student assessment. Each is designed to provide opportunity for *all* students and was developed with at least four curricular goals in

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MER Forum Announcements ..

MER Program at Annual Joint Mathematics Meetings

January 10 - 13, 2001

New Orleans, Louisiana

The list of speakers for the special session and the titles of their talks are on pages 8 and 9. Please consult the official program for all locations. Tickets for the MER Banquet may be purchased when registering for the meeting.

Wednesday, January 10

Special Session on *Mathematics and Education Reform*

Session I 8:00 -10:55 AM

Session II 2:15-6:05 PM

Thursday, January 11

Special Session on *Mathematics and Education Reform*

Session III 8:00-11:50 AM

Session IV 1:00-3:20 PM

MER Banquet 6:00 - 9:30 PM



PROGRAM IN MATHEMATICS FOR YOUNG SCIENTISTS (PROMYS)

Boston University, July 1 to August 11, 2001

PROMYS offers a lively mathematical environment in which ambitious high school students explore the creative world of mathematics. Through their intensive efforts to solve a large assortment of unusually challenging problems in number theory, the participants practice the art of mathematical discovery — numerical exploration, formulation and critique of conjectures, and techniques of proof and generalization. More experienced participants may also study algorithms, geometry and topology, or combinatorics. Problem sets are accompanied by daily lectures given by research mathematicians with extensive experience in Professor Arnold Ross's long-standing Summer Mathematics Program at Ohio State University. In addition, a highly competent staff of 15 college-aged counselors lives in the dormitories and is always available to discuss mathematics with students. Each participant will meet with professional mathematicians several times per week for problem-solving and open-ended explorations. Special lectures by outside speakers offer a broad view of mathematics and its role in the sciences.

PROMYS is a residential program designed for 60 ambitious high school students entering grades 10 through 12. Admission decisions will be based on the following criteria: applicants' solutions to a set of challenging problems included with the application packet; teacher recommendations; high school transcripts; and student essays explaining their interest in the program.

The approximate cost of room and board is \$1500. Books may cost an additional \$100. There is no cost to students for tuition thanks to the support of our sponsors. Financial aid is available. PROMYS is dedicated to the principle that no student will be unable to attend because of financial need.

PROMYS is directed by Professor Glenn Stevens. Application materials can be obtained by writing to PROMYS, Department of Mathematics, Boston University, 111 Cummington Street, Boston, MA 02215, by e-mail at promys@math.bu.edu, by visiting our website at <http://www.promys.org/>, or by calling (617) 353-2563. Applications will be available in February and accepted from March 1 until June 1, 2001.

... news briefs ... updates ...

The Boston College Mathematics Case Studies Project (BCCase) Workshop for Faculty

How can we develop the teaching skills of mathematics graduate students so that they can better meet their responsibilities as TAs and instructors? The Boston College Mathematics Case Studies Project (BCCase) addressed this question at a summer workshop for faculty involved in TA development held in Exeter, New Hampshire this past June. The workshop was designed to introduce such faculty to the case study method of teaching, and in particular to the set of case studies under development by BCCase for use by mathematics graduate programs. Participants included faculty and graduate students from Boston College, Boston University, Brown University, Cornell University, Harvard University, Oklahoma State University, Rhode Island College, University of Arizona, University of Chicago, University of Illinois at Chicago, University of Michigan, University of New Hampshire, University of Oklahoma, and University of Washington.

Solomon Friedberg of Boston College, principal investigator of the BCCase project, led off the workshop by describing the adaptation of the case study method, originally used in business and legal education, to teacher development; he then demonstrated the method by using a case study with the entire group. Following this, each faculty participant had the opportunity to lead a case discussion, and to receive peer feedback. In addition, a panel of graduate students discussed their experiences with the case studies, and two experienced case presenters, John J. Neuhauser, Academic Vice-President and Dean of the Faculty at Boston College, and Katherine Merseeth, Director of Research at the Harvard Children's Institute, shared their extensive experience and insights about leading case study discussions.

BCCase is developing a book of case studies that can be used to supplement existing graduate mathematics TA development programs or to form the core for new TA development programs. Funding for the project has been provided by The United States Department of Education, FIPSE grant P116B980015, with matching funds from Boston College. For more details please contact Prof. Friedberg at friedber@bc.edu.

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MASTER OF SCIENCE in NATURAL SCIENCES **Rensselaer Polytechnic Institute, June 25 to August 3, 2001**

A degree program leading to an MS degree in three successive summers is being offered to mathematics teachers by Rensselaer Polytechnic Institute. In addition to core courses in mathematics, science, and instructional technologies, additional courses in mathematics are offered which draw their content from visual mathematics, discrete mathematics and dynamical mathematics. The program emphasizes the use of instructional technology in the classroom as a theme in all three summers and relies heavily upon Rensselaer's international reputation as a leader in the development of these technologies. Partial tuition fellowships for all eligible applicants will permit the program to be offered at a rate well below Rensselaer's normal graduate tuition rate.

For more information and application forms contact Linda Fedigan at Center for Initiatives in Pre-College Education, CII 9217, Rensselaer Polytechnic Institute, Troy, NY 12180; (518) 276-6906; fax: (518) 276-2113; email: cipce@rpi.edu.

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MER Membership and Newsletter Subscription Information

To obtain an MER membership application form, send your mailing address to mer@math.uic.edu. Dues for the 2001 one-year individual membership, beginning January, 2001, are \$25. Individual dues for students are \$15. Membership in MER includes subscription to the bi-annual MER Newsletter and Special Issues, and access to the MER eNews, an electronic auxiliary.

AMS-MAA-MER SPECIAL SESSION on
Mathematics and Education Reform

SCHEDULE

Wednesday, January 10, AM

Mathematics Immersion Experiences in Professional Teacher Development Programs
Co-sponsored by EDC (Educational Development Center, Inc.)

8:00-8:20

Al Cuoco, EDC and Nancy Antonellis, EDC
Studying Mathematics to Improve Teaching

8:30-8:50

Kenneth M. Levasseur, University of Massachusetts Lowell
A Collaborative Approach to Professional Development through Problem Solving

9:00-9:20

Glenn H. Stevens, Boston University
PROMYS for Teachers: Learning in the Spirit of Exploration

9:30-9:50

Steven R. Benson, EDC and Karen J. Graham, University of New Hampshire
MST at UNH: A Mathematics Degree for Mathematics Teachers

Professional Master's Degrees in Mathematics Departments

10:00-10:20

Naomi D. Fisher, University of Illinois at Chicago
The Professional Master's Degree in Graduate Education

10:30-10:50

Thomas B. Schlumprecht, Texas A&M University
Professional M.S. Programs in Mathematics at Texas A&M

Wednesday, January 10, PM

Professional Master's Degrees in Mathematics Departments (continued)

2:15-2:35

Charles R. McCluer, Michigan State University
Hints on Establishing an Industrial Mathematics Program

2:45-3:05

William D. Stone, New Mexico Institute of Mining and Technology
Professional Master's Program in Math at New Mexico Tech

3:15-3:35

Dieter Armbruster, Arizona State University and Rosemary A. Renaut, Arizona State University
Professional M.S. Programs at Arizona State University, M.S. Statistics, M.N.S. High School Certification, and the M.N.S.

The Curriculum Foundations Project
Co-sponsored by CRAFTY (Calculus Reform And the First Two Years)

3:45-4:05

William H. Barker, Bowdoin College
The Curriculum Foundations Project: What Do Our Partner Disciplines Need?

4:15-4:35

Mary Ann Hovis, Lima Technical College, John C. Peterson, Chattanooga State Technical Community College, and Robert L. Kimball, Wake Technical Community College
Technical Mathematics for Tomorrow: Recommendations and Exemplary Programs

4:45-5:05

Thomas L. Moore, Grinnell College
Statistics: The Curriculum Foundation Workshop

AMS-MAA-MER SPECIAL SESSION on
Mathematics and Education Reform

SCHEDULE

5:15-5:35

Don Small, U.S. Military Academy and Kathleen G. Snook, U.S. Military Academy
Curriculum Foundations Workshop in Interdisciplinary Instruction at West Point

Thursday, January 11, AM

The Curriculum Foundations Project (continued), Co-sponsored by CRAFTY

8:00-8:20

Joan Ferrini-Mundy, Michigan State University
The Mathematical Preparation of Teachers: Issues for Mathematics Departments

Calculus: Evaluating Instructional Effectiveness
Co-sponsored by CRAFTY

8:30-8:50

Thomas W. Judson, University of Portland and Toshiyuki Nishimori, Hokkaido University
Are Students Learning the Necessary Concepts of Calculus? A Comparative Study of Japanese and American High School Students

9:00-9:20

Susan L. Ganter, Clemson University
Ten Years of Calculus Renewal: A Report on Evaluation Efforts and National Impact

Increasing the Number and Diversity of Teachers of Mathematics

9:30-9:50

Kenneth C. Millett, UC Santa Barbara
CAMP/CTFMS Mathematics Teaching Internships

10-10:20

Robert E. Megginson, University of Michigan
Project ENACT: An MAA Program to Increase the Number of Native American Mathematics Teachers

10:30-10:50

Philip C. Curtis, Jr., UC Los Angeles
UCLA Teacher Preparation Program in Mathematics

11:00-11:20

Maurice E. Gilmore, Northeastern University
Mathematically Enabling Students in the Boston Public Schools

11:30-11:50

Moderator: Millett; Panelists: Curtis, Gilmore and Megginson
Panel: Some Programs that Promote Mathematics Teaching Careers

Thursday, January 11, PM

Secondary Standards-based Curricula: Content and Implementation

1:00-1:25

James Fey, University of Maryland and Eric Robinson, Ithaca College
Improving Secondary School Mathematics with NSF-funded Curricula: What Are They Trying to Accomplish?

1:30-1:55

Sherry Fraser, Interactive Mathematics Project and Edward F. Wolff, Beaver College
The Interactive Mathematics Program

2:00-2:25

Solomon Garfunkel, COMAP and Jerry Lege, Columbia University
"Mathematics: Modeling Our World"—Reforming High School Curricula via a Modeling Immersion Approach

2:30-3:20

Moderator: Robinson; Panelists: Midge Cozzens, University of Colorado at Denver; Wolff; and Paul Zorn, St. Olaf College
Panel: Implementation Issues Surrounding New Directions in High School Mathematics

Mathematical Knowledge for Teaching

continued from page 3

determine which will be the most “orangey” and which will be the least “orangey.” The recipes are as follows:

Mix A: 2 cups concentrate, 3 cups water

Mix B: 1 cup concentrate, 4 cups water

Mix C: 4 cups concentrate, 8 cups water

Mix D: 3 cups concentrate, 5 cups water

One of the dangers of posing such a problem to students and discussing the solutions is that students will come up with many different solutions, some correct and some incorrect. If the teacher doesn't attempt to tie the solutions together, then some students, rather than failing to understand the single method the teacher demonstrated in a traditional class, may fail to understand six or eight methods demonstrated by different students.

Actual student work provides an opportunity to do this important work of teaching. (See Chappell, in press, for samples.) In analyzing such student work, prospective and practicing teachers can be encouraged to categorize solutions according to mathematical features. At first pass, the students' work seems quite diverse, using a variety of representations, including tables, pictures, fractions, decimals, and percents. Some students solve the problem by comparing the fractions $\frac{2}{3}$, $\frac{1}{4}$, $\frac{4}{8}$, and $\frac{3}{5}$. They find a common denominator or a common numerator or convert the fractions to decimals or percents. Other students compare $\frac{2}{5}$, $\frac{1}{5}$, $\frac{4}{12}$, and $\frac{3}{8}$, using the same methods to do the comparison. Other students don't use fractions at all, but perform analogous comparisons in tables or pictures.

Rather than organizing student solutions by the representation used, teachers might be asked to abstract from the particular representations to see bigger ideas. Consider the following scheme for organizing the solutions:

part-part strategies that compare concentrate to water or water to concentrate, and

part-whole strategies that compare concentrate or water to the whole recipe.

This scheme leads to the following mathematical question: *Will the part-part and part-whole solution methods always yield the same result? Why or why not? Can you prove it?*

The connections between the various representations are not obvious. Teachers might be asked to make the connections explicit and to compare the efficiency of various approaches.

For people who are concerned about teacher education, the question might be, What kind of mathematical knowledge and sensibilities are necessary to arrive at the above categorization and synthesis of solutions? What mathematical sensibilities are required to notice, for example, that a pictorial and a tabular solution are isomorphic because they both reduce the recipes to equivalent recipes that use one can of concentrate?

Conclusion

Improvements in mathematics teaching depend upon the extent to which teachers use mathematical knowledge in their teaching. Mathematics teacher education should help teachers make connections between students' knowledge and mathematical content. Yet this requires pedagogical knowledge that is ignored by the traditional separation between content and pedagogy. This essay suggests a promising approach, but much work remains to be done both to make this knowledge explicit and to create opportunities for teachers to acquire it.

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Focusing on Secondary Mathematics

continued from page 5

mind: to create mathematically literate citizens, to provide mathematics needed for the workplace, to provide a foundation for further study of disciplines that require significant amounts of mathematics (including mathematics itself), and to create independent mathematical learners.

These programs function very differently from most of the high school curricula that were in use in 1989 (and are still in many schools today). One cannot discern these differences simply from a glance at a table of contents or a few textbook page flips. Indeed, one of the reasons for providing opportunities such as the MER sessions at the Joint Meetings is that the descriptions and underpinnings of these differences cannot and should not be condensed into a few lines of text. Some of the reasons for the new approaches in these curricula are subtle and complex.

There are some obvious and common characteristics that these programs share. Each curriculum is “integrated” in the sense that each year contains content from several areas in mathematics, usually including algebra and functions, geometry, probability and statistics, and discrete mathematics. Most often these topics not only appear in the same year, they appear in the same unit when it makes sense to develop concepts and techniques from different disciplines in order to solve a particular problem. One of the reasons the curriculum developers chose to integrate topics is that it helps to strengthen connections among mathematical disciplines and unify the subject. Moreover, not only does this approach expand the range of problems available for students to study, it accurately depicts mathematics of the 21st century. That is, many real-world problems and advanced pure mathematical problems involve solutions that incorporate concepts and techniques from several areas within mathematics.

A second common feature of the curricula is that the *students* are required to do mathematical reasoning. Here, mathematical reasoning includes logical deduction and mathematical proof, but it also includes other aspects of mathematical thinking such as having students search for patterns, explore examples, form conjectures, verify special cases, abstract properties, draw analogies, generalize results, choose strategies and mathematical representations for solving problems, create mathematical models of given situations, formulate and analyze algorithms and formulas, and pose problems. This, too, is closer to what real mathematics is about.

These curricula may challenge assumptions about the teaching and learning of mathematics. For example, many traditional texts “cover” a topic by isolating it and discussing it in an uncluttered, abstract form. Then several “template” exercises are presented. Students are then expected to round out their understanding of the topic by completing numerous practice exercises, most

quite similar to the template problems, with perhaps a few applications at the end. These curricula often do the reverse. Students develop understanding of mathematical concepts first intuitively, within the context of a problem. They gradually deepen this understanding throughout the unit or longer, moving from the concrete to the abstract—often in settings where the topic is fastened to the study of other topics.

Conclusion

Understanding between the mathematics faculty in secondary and higher education needs to be strengthened. Information gathering is a beginning, but we need to go beyond just knowing what mathematical knowledge and abilities secondary students bring to the college and university classroom. There is more commonality in improvements that are going on at the two educational levels than one might guess. There is much to be learned and much to be gained by all if we work more closely together in collegial settings. Grounded in sufficient and accurate information, it is possible for professionals across the educational levels to speak about these commonalities with one voice. That voice will help efforts to improve mathematics education at both levels and just might raise the understanding of an educated citizenry with regard to the nature of mathematics itself.

Footnotes

1. National Research Council. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Washington D.C.: National Academy Press, 1989.
2. National Commission on Excellence in Education. *A Nation at Risk: The Imperative for Educational Reform*. Washington, D.C.: U.S. Government Printing Office, 1983.
3. National Research Council. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Washington D.C.: National Academy Press, 1989, pg.1
4. *Ibid.*, pg. 96

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Membership Announcements

2001 MER Membership

Renewal notices for MER membership will be coming to you soon. As in the past two years, MER membership includes subscription to the MER newsletters and access to the MER eNews, an electronic auxillary. We at MER look forward to your continued participation in the MER Forum.

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Year 2000 MER Newsletters are still available to interested persons. Non- MER members wishing to obtain the three MER Newsletters, published in 2000 including the Special Issue *Minority Mathematicians: Who is Responsible?* may purchase them by sending the \$25 year 2000 membership fee to The MER Forum, University of Illinois at Chicago, Department of MSCS (M/C 249), 851 S. Morgan St., Chicago, IL 60607-7045. For questions regarding these issues you may email mer@math.uic.edu.

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MER Special Session on *Mathematics and Education Reform*

Annual Joint Mathematics Meeting in New Orleans

A schedule for the MER Special Session on Mathematics and Education Reform appears on pages 8 and 9 of this newsletter. A schedule and abstracts for the individual talks at these sessions are available at the MER Homepage, <http://www.math.uic.edu/MER/>.

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More Announcements on pages 6 and 7.

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