Newsletter

Mathematicians and Education Reform ...Forum...

Fall 1999 Volume 12 Number 1

Concept & Computation: Role of Curriculum

by John Baldwin, University of Illinois at Chicago and
Kathy Kessel, University of California, Berkeley

In *Knowing and Teaching Elementary Mathematics*, Liping Ma considers Chinese and U.S. teachers' understanding of topics in elementary mathematics.[1] In this article we will use some of the tools developed by Ma to discuss what might constitute such an understanding of a slightly more advanced topic: integers. This includes integer arithmetic and integers on the number line.

In talking about how to teach a particular topic, Ma and the Chinese teachers she interviewed discussed its connections with other topics. Such connections occurred both prior and later in the curriculum, and were made directly between topics and indirectly via general principles. The Chinese teachers discussed them as parts of "knowledge packages" for a given topic—conceptual and procedural topics that support and are supported by the learning of the topic. One teacher described it as a way of thinking in which one sees topics group-by-group rather than piece-by-piece:

. . . you should see a knowledge "package" when you are teaching a piece of knowledge. And you should know the role of the present knowledge in that package. You have to know that the knowledge you are teaching is supported by which ideas or procedures, so your teaching is going to rely on, reinforce, and elaborate the learning of these ideas. (Ma, 1999, p. 18)

What might knowledge packages be for the integers? Obviously it depends on exactly how you delineate the topic and we'll see below that there are several extant variations. For a particular interpretation one can ask:

What topics support the learning of this topic?
What topics are supported by knowledge of this topic?

The answers to these questions constrain the placement of the topic in the curriculum. In Ma's book information about the Chinese national curriculum and textbooks occurs mainly to explain statements in the teachers' interviews. In contrast to Ma's focus on interviews with teachers, this article proceeds from examples of textbooks, curricula, and standards to questions about U.S. conceptions of a particular topic—the integers—and its connections with other topics, and more generally to suggest how such questions might be used in thinking about curricula.

At what grades do negative numbers occur?

Examples of U.S. textbooks, standards, and guidelines suggest that negative numbers have an uncertain relationship with the rest of the curriculum. Negative numbers may first occur in some form in any elementary grade. For example, an experimental curriculum of the 70s (CSMP) begins work with integer addition in 1st grade. The recently published *Math Trailblazers* discusses placement of marks on a line in 3rd grade and negative numbers on the number line in 4th grade. The reform textbook series *Mathland* introduces negative numbers in 6th grade, allocating a week to the negative numbers on

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from the editor

Knowing and Teaching Elementary Mathematics by Liping Ma, published by Lawrence Erlbaum Associates, Inc., 1999, has become a phenomenon in mathematics education and mathematics circles. Liping's *knowledge packages* and *Profound Understanding of Fundamental Mathematics* resonate with notions held dear by mathematics educators and mathematicians. Knowledge packages position concepts and processes within a connected structure of ideas, rather than list them in isolation from one another. Profound Understanding of Fundamental Mathematics is "an understanding of the terrain of fundamental mathematics that is deep, broad, and thorough." A significant indication of a teacher's possessing such mathematical understanding is his or her fluency in developing multiple models for the instruction of a concept.

Concept and Computation: Role of Curriculum (page 1) by John Baldwin and Kathy Kessel builds on the concept of knowledge packages to consider the teaching of negative numbers. Their investigation of a variety of curricula and American published texts shows an almost bewildering range of approaches. What emerges, however, is a better appreciation of the complexity of teaching that is grounded in meaningfully connected ideas.

It's good to keep in mind that *mathematicians love mathematics* when "doing education" with mathematicians. I have observed again and again how mathematicians latch onto even a kernel of mathematics, ponder it, and then return it having reformulated and linked it to a larger body of mathematics. *Exploring Math with Elem School Teachers* (page 3) by Paul Sally is a window into the mind of one mathematician as he moves back and forth between thinking about mathematics and thinking about teaching mathematics.

One of the ongoing challenges to the mathematics community is to increase the number of minority PhDs. *Lessons at the University of Maryland* (page 12) by Duane Cooper explains how several factors have been essential to producing a community of black mathematics graduate students at Maryland. Although Cooper also shares some cautionary lessons, the Maryland model should be studied for adaptation on other campuses.

Naomi D. Fisher

The **Mathematicians and Education Reform (MER) Forum** seeks the effective participation of mathematicians in mathematics education reform at the K-12, undergraduate, and graduate levels, and the recognition of the importance of these efforts to the well being of the mathematics community. The MER Forum envisages the pursuit of educational reform through informed discussion of educational issues, thoughtful responses to changing educational conditions, and the promotion of exemplary programs. The creation and support of a network of mathematicians with a sustained commitment to mathematics education is central to this vision.

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Notes on Mathematical and Teaching Ideas

Exploring Math with Elem School Teachers

by Paul J. Sally, Jr., University of Chicago

Introducing and Exploring the Problem

At the May 8-10, 1998 MER workshop *on Developing Leadership in Elementary and Middle School Education* in Chicago, Marj Enneking of Portland State University introduced a problem about lattice polygons in the plane that she had used in a class for preservice teachers. Marj drew a 6 X 6 lattice and posed the question "How many different size squares can be embedded in this lattice?" Marj explained that she had presented the problem to her students by way of a geoboard. Several of her colleagues had noticed the problem in passing by her class, got interested, and began working on it. It turned out to be the kind of problem that challenged and stimulated students and colleagues alike.

I have studied lattice polygons in the plane, and, more generally, lattice polyhedra in n-space for some time, but I had never encountered this problem before Marj stated it. The problem fits into the general category of questions such as, "Which regular polygons can be embedded in the plane as lattice polygons?"

One of the more interesting facts about lattice polygons is Pick's theorem. Pick's theorem is often stated in the following way. If P is a lattice polygon, we denote by $L(P)$ the number of lattice points inside and on the boundary of P, by $B(P)$ the number of lattice points on the boundary of P, and by $A(P)$ the area of P. Then

$$A(P) = L(P) - 1/2\, B(P) - 1.$$

This is often used to compute the area of lattice polygons simply by counting lattice points. Note that this gives us the fact that the area of a lattice polygon is a half-integer. Thus, for example, an equilateral triangle cannot be embedded as a lattice polygon because the area of an equilateral triangle is $s^2\sqrt{3}/4$ where s^2 is the square of the side-length of the triangle. In the case of a lattice triangle, s^2 is an integer, but certainly, $s^2\sqrt{3}/4$ is not a half-integer. The problem of embedding any regular n-gon, $n \geq 5$, as a lattice polygon is an intriguing one.

Now, back to the original question. If we draw a lattice square in the plane, we can assume without any loss of generality that the vertices are $(0,0)$, (a,b), $(-b,a)$, and $(a-b,a+b)$. (You can picture the square imbedded in a larger square in standard position and with side of length $a+b$.) The area of this square is a^2+b^2, so the answer to the question "What areas can occur for lattice squares?" is any positive integer which can be written as the sum of the squares of two nonnegative integers. This is answered by Fermat's two squares theorem, which is stated as follows:

If n is a positive integer, write $n = m^2k$ where k is square-free. Then n can be written as the sum of squares of two nonnegative integers if and only if the only odd primes contained in the factorization of k are of the form $4j+1$.

An Instructional Application

I tried these ideas with a class of elementary school teachers in summer 1999. I first encouraged them to look at Marj Enneking's problem with lattices of different sizes. Working in groups, the teachers came up with lattice squares whose areas were 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, and 20. Of course they began to ask questions about what numbers could occur and how one could determine that. I gave a hint.

$1 = 0^2 + 1^2$
$2 = 1^2 + 1^2$

I then challenged them to write 3 as the sum of two squares. They quickly agreed that it couldn't be done. So I said, continue the pattern, and they generated a table that looked like this.

1	$0^2 + 1^2$	11	NO
2	$1^2 + 1^2$	12	NO
3	NO	13	$2^2 + 3^2$
4	$0^2 + 2^2$	14	NO
5	$1^2 + 2^2$	15	NO
6	NO	16	$0^2 + 4^2$
7	NO	17	$1^2 + 4^2$
8	$2^2 + 2^2$	18	NO
9	$0^2 + 3^2$	19	NO
10	$1^2 + 3^2$	20	$2^2 + 4^2$

After more experimentation, the teachers discovered that $25=0^2+5^2 = 3^2+4^2$, and further discussion ensued about the number of ways in which a positive integer can be written as a sum of two squares. I didn't pursue this further, although there is an interesting answer to this question. The teachers found this a thoroughly entertaining and instructive problem relating ideas in geometry and number theory. Thanks Marj! ■

Concept &Computation: Role of Curriculum

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the number line and their addition and subtraction (Unit 5, week 3, Integer Investigations). In contrast, the Algebra Project devotes a supplemental module spread over a semester of 6th grade to a thorough introduction of the number line and addition and subtraction of positive and negative numbers.[2]

The 1989 NCTM *Standards* urge the study in grades 5–8. Somewhat more ambitiously and precisely the recently adopted California Mathematics Standards lays out a progressive study of the negative numbers beginning in 4th grade. Negative numbers and the number line are introduced the 4th grade, but arithmetic with negative numbers doesn't occur until 6th grade. According to these standards students should:

"Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in 'owing')" in 4th grade.

"Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers" in 5th grade.

"Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line" and "Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals)" in 6th grade.

In contrast, the Japanese national curriculum introduces negative number topics much later. Although "number line" is mentioned in the list of terms and symbols for 3rd grade, it does not include negative numbers. The first mention of negative numbers occurs as part of the three objectives for the equivalent of 7th grade (grade 1 of lower secondary school) and is further elaborated as one of the seven main mathematics topics for the grade:

> *To enable students to understand the meaning of positive and negative numbers, and to compute with those numbers according to four fundamental operations.* (Research Center for Science Education, 1989, p. 25)

The two approaches illustrated here (several short treatments vs. one long treatment) typify the results of the TIMSS curriculum study: On average, a given topic occurs more briefly and more often in U.S. textbooks and curricula than in those of other countries (Schmidt, McKnight, & Raizen, 1997).

Where do negative numbers occur in textbooks?

Can a gradual treatment such as that outlined by the California Standards prepare students for integer arithmetic? Or will such an approach suffer from the fragmentation discussed by Mayer, Sims, and Tajika (1995)? They studied lessons on the addition of signed numbers in three Japanese and four U.S. textbooks and found in "three U.S. books, material on addition and subtraction was in the same chapter as solving equations and coordinate graphing of equations; in another it was taught in a chapter that included units of measurement, mixed numbers, and improper fractions" (p. 456). A similar question concerns *Mathland*: in the 6th grade book, one week on integer addition and subtraction and the number line is preceded by a week on prime numbers and followed by a unit about areas of polygons.

In *Japanese Grade 7 Mathematics* the number line (with both positive and negative numbers), integer addition and subtraction, and integer multiplication and division occur in a chapter entitled "Positive and Negative Numbers." This is followed by "Letters and Expressions," a chapter about expressing situations in terms of variables and manipulating expressions that include variables.

Meanings and models for negative numbers

Part of the variability in occurrence of negative number topics in U.S. curricula may be due to different ways in which these topics are construed. Ball (1993) points out that negative numbers have at least two important aspects: they can be used to represent an amount of the opposite of something, and to represent a location relative to zero. Generalizing this slightly, the various models for negative numbers can be grouped into two categories: negatives represent an amount of the opposite of something (an additive inverse), or a location (direction and magnitude). CSMP, the curriculum used by Ball's school district illustrates negative numbers with a scenario involving magic and regular peanuts. If a magic and a regular peanut were in a pocket at the same time both disappeared. Concerned about fostering magical notions about mathematics, Ball rejected the peanut model for one involving paper people who went several floors up or down on a building with a ground floor of 0 (see Ball, 1993 for details). Part of her motivation for choosing this model was that it was a positional model like the number line, but might allow modeling of integer addition and subtraction (it turned out that Ball's third graders had difficulty interpreting 6 + (–6) in terms of elevator rides).

Math Trailblazers introduces Mr. O, the origin, in 3rd grade. In this short section, students mark places to the left and right and front of and back of Mr. O, but negative numbers do not literally appear. There is a six-page section introducing negative numbers on the number line in grade 4; examples and problems include temperature, altitude, and bank balance. In grade 5, there is a full unit (2–3 weeks) devoted to maps and coordinates. This unit passes from concrete problems of maps to more abstract tranformations on the plane (what happens when an "L" is flipped across the *y*-axis?). Thus, these materials present a gradual introduction to the location aspect of negative numbers without any real connection to the arithmetic or "opposite" aspect.

In the grade 6 *Mathland*, the number line and integer addition and subtraction occur in the same week, but the two aspects of location and opposite do not seem connected in the activities or the text. The first day is devoted to location on the number line, the next three days to integer addition, and the last day to integer subtraction. Integer addition is introduced with pink cubes representing positive numbers and green cubes representing negative numbers. Pairs with both colors "cancel out so they equal 0." Students use this model for calculating sums of integers, and later for integer subtraction. According to the teacher's manual, the "key ideas" of this unit are: "Just as the number line extends infinitely in the positive direction, it also extends infinitely in the opposite, negative direction"; and "Generalizations about the outcomes when integers are added or subtracted can be made based on relationships between the numbers" (p. 154). These are not connected with each other explicitly in the text and do not seem connected in the activities.

The 7th grade U.S. textbook examples discussed by Mayer, Sims, and Tajika suggest that connecting the two aspects of negative numbers may not have been an emphasis of all the textbook authors. Two of the models focused on opposite: a beaker containing positive and negative charges, and scenarios involving matter and antimatter. The other two models had elements of both opposite and location: Temperature and ratings (the ratings calculation was illustrated with a number line, suggesting a connection with location as well as opposite).

The models for integer addition presented in the three Japanese textbooks all illustrate opposite and location. However, location in two of the three models is not relative to zero, but to a given starting point. For example, *Japanese Grade 7* uses changes in water level in a tank. The initial point is "initial water level" on an unlabeled scale, a positive number corresponds to an increase in water level and a negative number corresponds with a decrease in water level. (Variations of this example are used later in the text to illustrate integer multiplication and division, e.g., if water flows in at 10 cm/min, how much higher is the water level 3 minutes later and how much lower was the water level 3 minutes earlier?) Another text uses the example of walking east or west from a given point A on an unlabeled road.

In devising the Algebra Project for grade 6, Robert Moses analyzed the role of positive and negative numbers in the differing meanings of subtraction for natural numbers and integers. The Algebra Project first introduces the notion of displacement. This is a deep treatment, motivated for students by a trip on public transit, and with an associated exploration of the notion of equivalence—vectors are equivalence classes. Then the students explore the difference between the old subtraction metaphor, "take away" and the new, "compare."

Possible pieces of a knowledge package

Moses's analysis suggests that a necessary piece of knowledge for learning negative numbers is the idea of subtraction as "compare" as opposed to "take away." First and second grade U.S. textbook word problems often focus on the "take away" interpretation of subtraction and the associated solution procedure of "sum – addend = [answer]" (Fuson, 1992). The associated strategy (known as "counting down") is often more difficult for students than strategies associated with different interpretations. A subtraction, say, 15 – 8 = ? can also be viewed as "How much *more* is 15 than 8?" which suggests the solution strategy of "counting up" from 8.[3]

The Algebra Project also suggests that students need to understand subtraction as the inverse of addition—a piece of knowledge that also occurs in the knowledge package for subtraction with regrouping (Ma, 1999, p. 19). Moses, Kamii, Swap, and Howard (1989) explain,

Most [U.S.] algebra texts introduce subtraction as a transformed addition problem. Students are asked to think of subtraction (3 – (–2)) = +5) as "adding the opposite" or "finding the missing addend" (3 – ? = 5). . . . The problem is compounded because [U.S] students have overlearned "take-away" as the concept underlying subtraction. In algebra, "take-away" no longer

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MER Forum Announcements ..

MER Program at Annual Joint Mathematics Meetings

January 19 - 22, 2000 **Washington DC**

The list of speakers for the special session and the titles of their talks are on pages 8 and 9. Please consult the official program for all locations.

Special Note: The usual Thursday night MER Banquet will not take place this year. MER will be represented at the Opening Banquet on Tuesday evening, as will the other organizations that regularly schedule evening banquets.

Wednesday, January 19

Special Session on *Mathematics and Education Reform*

Session I	**8:00 -10:55 AM**
Session II	**2:15-6:05 PM**

Thursday, January 20

Special Session on *Mathematics and Education Reform*

Session III	**8:00-11:00 AM**
Session IV	**1:00-3:30 PM**

Themes in the 2000 Special Session

The 2000 special session on Mathematics and Education Reform, organized by William Barker, Bowdoin College, Jerry Bona, University of Texas at Austin, Naomi Fisher, University of Illinois at Chicago, and Kenneth Millett, University of California Santa Barbara, is organized into seven themes. Collaborations with other mathematics organizations and individuals in selecting the topics and identifying speakers make the session a forum for highlighting the diversity of mathematics education issues within the mathematics community. The issues for the 2000 special session are:

- *Teaching Undergraduate Mathematics: Several Research Perspectives*, co-sponsored by ARUME, serves as an introduction to the questions and findings being investigated in mathematics education research at the undergraduate level.

- *Minority mathematicians: Who is responsible?* is intended to raise questions as well as provide examples of how some departments are attracting minority students into mathematics.

- *Project NExT (New Experiences in Teaching),* co-sponsored by Project NExT, is an update of what is happening to young mathematics faculty several years after their entrance into the profession as seen by Project NExT leaders, a department chair, and especially, several young faculty.

- *Mathematicians, the NCTM, and the Standards 2000 Project*, co-sponsored by NCTM, features a panel of mathematicians who are directly involved in the writing of the NCTM's Standards 2000, as authors and as the Executive Director of NCTM. The panel will explore the roles of mathematicians in working with the K-12 mathematics community on curricular and instructional reform at the K-12 level.

- *Notes on new projects* offers a look at two different areas: getting middle grade girls active in a mathematics project, and a new approach to preparing mathematics teaching assistants.

- *A Facilitated Dialog on Teaching Reform*, co-sponsored by CRAFTY, is a first step in bringing together proponents of different views on instructional reform as proposed by William Davis in last year's special session.

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- *Some problems besetting mathematics education from Kindergarten on up and ways for university faculty to contribute in solving them*, co-sponsored by AWM, is organized to present suggestions of how individual mathematicians can make effective contributions in K-12 mathematics education.

The special session co-organizers would like to extend their appreciation to the individuals who served as special co-organizers for sections of the program:

William Davis, The Ohio State University, co-organizer of the panel *A Facilitated Dialog on Teaching Reform Issues.*

Raymond Johnson, University of Maryland, co-organizer of the *Minority Mathematicians* section.

Annie Selden, Tennessee Technological University, co-organizer of the *Teaching Undergraduate Mathematics* section.

Christine Stevens, St. Louis University, co-organizer of the *Project NExT* section.

Virginia Warfield, University of Washington, co-organizer of the *Some problems besetting mathematics education...and ways for university faculty to contribute in solving them* section.

PROGRAM IN MATHEMATICS FOR YOUNG SCIENTISTS (PROMYS)
Boston University, July 2 to August 12, 2000

PROMYS, directed by Professor Glenn Stevens of Boston University, is a residential program in which ambitious high school students entering grades 10 through 12 can explore the creative world of mathematics. Through their intensive efforts to solve a large assortment of unusually challenging problems in Number Theory, the participants practice the art of mathematical discovery — numerical exploration, formulation and critique of conjectures, and techniques of proof and generalization. More experienced participants may also study algorithms, geometry and topology, or combinatorics. For further information on programming, application requirements and procedures, cost and financial aid write to PROMYS, Department of Mathematics, Boston University, 111 Cummington Street, Boston, MA 02215, or e-mail at promys@math.bu.edu , or visit our website at http://math.bu.edu/people/promys, or call (617) 353-2563. Applications will be available in February and accepted from March 1 until June 1, 2000.

Masters Program for Mathematics Teachers
Rensselaer Polytechnic Institute, June 26 - August 4, 2000

A degree program leading to an MS degree in three successive summers is being offered to mathematics teachers by Rensselaer Polytechnic Institute. In addition to core courses in mathematics, science, and instructional technologies, courses in mathematics are offered which draw their content from visual mathematics, discrete mathematics and dynamical mathematics. The program emphasizes the use of instructional technology in the classroom and relies heavily upon Rensselaer's international reputation as a leader in the development of these technologies. Partial tuition fellowships for all eligible applicants will permit the program to be offered at a rate well below Rensselaer's normal graduate tuition rate. For more information and application forms contact Linda Fedigan at *Center for Initiatives in Pre-College Education, CII 9217, Rensselaer Polytechnic Institute, Troy, NY 12180; (518) 276-6906; fax: (518) 276-2113; email: cipce@rpi.edu.*

MER Membership and Newsletter Subscription Information

To obtain an MER membership application form, send your mailing address to mer@math.uic.edu. Dues for a one-year 1999-2000 individual membership, beginning December, 1999, are $25. Individual dues for students are $15. Membership in MER includes subscription to the bi-annual MER Newsletter and access to the MER eNews, an electronic auxiliary.

AMS-MAA-MER SPECIAL SESSION on

Mathematics and Education Reform

SCHEDULE

Wednesday, January 19, AM

Teaching Undergraduate Mathematics: Several Research Perspectives
Co-sponsored by the Association for Research in Undergraduate Mathematics Education (ARUME)

8:00-8:20
Marilyn Carlson, Arizona State University
A Survey of the Function Literature: What Have We Learned?

8:30-8:50
Barbara Edwards, Oregon State University
Undergraduate Mathematics Students' Understanding of the Role of Definitions and the Consequences of These Understandings on Students' Ability to Use Definitions in a Mathematically Acceptable Way

9:00-9:20
Ed Dubinsky, Georgia State University
Effects on students learning topics in collegiate mathematics of pedagogy based on APOS Theory, using cooperative learning, students writing computer programs and in-class problem solving

Minority Mathematicians: Who is responsible?

9:30-9:50
William Velez, University of Arizona
Minorities are Invisible in Mathematics

10:00-10:20
Etta Falconer, Spelman College
Attracting Undergraduate Minorities To Mathematics

10:30-10:50
David Manderscheid, University of Iowa
Increasing the Number of Minority PhDs in Mathematics

Wednesday, January 19, PM

Project NExT (New Experiences in Teaching)
Co-sponsored by Project NExT

2:15-2:35
Christine Stevens, St. Louis University, Joseph Gallian, University of Minnesota-Duluth, and Aparna Higgins, University of Dayton
Mathematics departments, new faculty, and Project NExT

2:45-3:05
Michele Intermont, Kalamazoo College
From One Community to the NExT

3:15-3:35
Douglas Ensley, Shippensburg University of Pennsylvania
New Experiences with Tenure

3:45-4:05
Jim Lewis, University of Nebraska, Lincoln
Project NexT – A Chair's View

Mathematicians, the NCTM, and the Standards 2000 Project
Co-sponsored by the National Council of Teachers of Mathematics (NCTM)

4:30-6:00
Kenneth Millett, University of California, Santa Barbara, Alfred Manaster, University of California, San Diego, James Sandefur, Georgetown University, John Thorpe, NCTM, and Philip Wagreich, University of Illinois at Chicago

Panel on
Mathematicians, the NCTM, and the Standards 2000 Project

AMS-MAA-MER SPECIAL SESSION on

Mathematics and Education Reform

SCHEDULE

Thursday, January 20, AM

Notes on new projects

8:00-8:20
Sarah Berenson, North Carolina State University, Tiffany Barnes, North Carolina State University, Laurie Cavey, North Carolina State University, and Virginia Knight, Meredith College
Girls on Track: Middle Grade Girls Modeling Community Problems – An Experiment in Progress

8:30-8:50
Solomon Friedberg, Boston College
Training Mathematics TAs Using Case Studies

9:00-9:20
Discussion

A Facilitated Dialog on Teaching Reform Issues
Co-sponsored by Calculus Reform And the First Two Years (CRAFTY)

9:30-11:00
William Davis, The Ohio State University, Steven Krantz, Washington University, Alan Tucker, SUNY Stony Brook, Steven Zucker, Johns Hopkins University, and Lauren McGarity, Alternative Solutions, Inc.

Panel on
A Facilitated Dialog on Teaching Reform Issues

Thursday, January 20, PM

Some problems besetting mathematics education from Kindergarten on up and ways for university faculty to contribute in solving them
Co-sponsored by the Association for Women in Mathematics (AWM)

1:00-1:20
Gail Burrill, National Academy of Science
Learning to make a difference

1:30-1:50
Virginia Warfield, University of Washington
Can one person make a difference? Yes!

2:00-2:20
Bernice Sandler, National Association for Women in Education
Mentoring: Myths and Realities, Dangers and Responsibilities

2:30-2:50
Shirley Malcom, American Association for the Advancement of Science (AAAS)
Rethinking K-12 Mathematics Education

3:00-3:30
Discussion

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has a straightforward application to subtraction. Within a couple of months of beginning algebra, students confront subtraction statements which have no discernable content, have only indirect meaning in relation to an associated addition problem, and are not at all obvious. (p. 434)

In 6th grade, one aspect of the Algebra Project is compensating for a mistaken emphasis in the first two years of school.

Is this focus necessary for learning about negative numbers—or only for learning algebra? Or do the two rely on some of the same pieces of knowledge? After working with students who had difficulty with algebra, Moses concluded that "the heart of the problem lay in their concept of number" (Moses et al., 1989, p. 432): In arithmetic students focus on magnitude, but in algebra students must be able to focus on magnitude and direction—"Which way?" as well as "How many?"

This suggests that negative numbers may be an important precursor for algebra, acting as what the Chinese teachers called a "concept knot"—a single concept that ties several important ones together (Ma, 1999, p. 78). Choosing models for the integers that tie magnitude and opposite together may be particularly important, for not only may they support students' coordination of both aspects of the integers, but may in turn support later learning of algebra.

Perhaps this is part of the reason the chapter on integers occurs in *Japanese Grade 7* immediately before the chapter on Letters and Expressions. Moreover, the two chapters have other themes in common: commutative, associative, and distributive laws; connections between lengths marked on a diagram and expressions involving numbers or symbols. The latter may support learning about functions and graphs (a later topic for grade 7 Japanese students).

The possible knowledge package sketched above is associated with a curricular approach involving one long treatment of the integers. However, the question of whether to take an in-depth or incremental approach seems less important than understanding which topics support and are supported by knowledge of the integers. Both teachers and curriculum writers need to consider such knowledge packages in determining where to place the study of various topics and how to establish connections among topics.

Acknowledgements

We would like to thank especially Bill Crombie, Cathy Kelso, Danny Martin and Miriam Gamoran Sherin among the many who supplied relevant information and comments on earlier versions of this article.

Footnotes

[1]All of the Chinese and almost all of the American teachers had experience teaching in only grades 1–6. (The first author mistakenly asserted in the May issue of the MER *Newsletter*: "The Chinese teachers were spread among grades K–8. . . ." The Chinese teachers in Ma's study were elementary teachers and "elementary school" in China is grades 1 to 6.)

[2]A technical note may be in order here. One can't assume that mathematics lessons correspond directly with what's in the textbook. In Japan and China, however, teachers are expected to elaborate on what's in the textbook, but they do follow the order in which topics are presented (see e.g., Ma, 1999 or National Institute on Student Achievement, Curriculum, and Assessment, 1998). In the U.S., part of the role of the teacher is to choose topics from the textbook, perhaps with the assistance of state and district guidelines (Schmidt, McKnight, & Raizen, 1997).

[3]Some elementary school projects and programs, for example the Cognitively Guided Instruction Project (Carpenter, Fennema, & Franke, 1996) and *Math Trailblazers* encourage many different addition and subtraction strategies. Teachers are aware of different possible strategies for solving the same problems and include problems associated with different interpretations of subtraction. ■

Editor's Note:
The complete list of references for the above article is available on request from the Editor of the MER Newsletter.

Also, the article, including the complete list of references, may be found on the web at http://www.math.uic.edu/~jbaldwin/mathed.html.

Lessons at the University of Maryland
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mathematics faculty at Maryland, students and prospective students realize that this department has two *more* than most universities. Both professors have been instrumental in attracting Black students to graduate study here.

By its existence, the community of Black mathematics graduate students seems to undergo perpetual regeneration. It attracts new students each year who are heartened to find a graduate program in which they feel they belong.

Lesson 3: Academic and Social Peer Support Are Vital

Academic and social peer support are vital for many students to survive the hard work of a mathematics Ph.D. program, especially the transitional first year. A large majority of the Black graduate students view peer support as special and important, which comes largely, though not exclusively, from their fellow Black students, primarily in academic collaboration and social support.

The potential for academic collaboration among Black students is a unique feature of having a large cohort of Black graduate students. At other institutions, where Black students are often scattered among different entry classes they are not taking key courses and preparing for major examinations at the same time. In contrast, at Maryland many students take advantage of the opportunity to enroll in classes together. While being "the only one" in a class is the national norm for Black mathematics graduate students, at Maryland about half of the Black students, including the advanced students, report that there have been two or more Black students in all or almost all of their courses at the university.

All of the students have participated in study groups, which may be all Black or multiracial. The students have mixed feelings about group study; many prefer to study alone. Still, most of them participate in groups, regardless of whether or not other Black students are taking the class.

Lesson 4: Mentorship is Important

Mentorship before and during graduate school is important to influence and academically prepare students to pursue the Ph.D. and to facilitate progress towards the degree. Not all of the Black mathematics graduate students feel they have benefitted greatly from mentoring, but many do and they speak quite passionately about it.

More than half of the Maryland Black graduate students report that someone from their undergraduate institution influenced their decision to pursue a doctorate in mathematics. In all but one instance, the influential faculty or staff member was either Black or from a Black college.

Mentors are important during graduate study, as well. Some undergraduate professors continue to mentor their former students. The department's Black faculty and postdoctoral fellows, some non-Black faculty members, other campus personnel, and mathematicians from outside this university also act as mentors. The primary assistance that mentors give students is on-going advice and encouragement.

Help with subject matter most commonly comes from senior graduate students, who often assume two roles: peer supporter and mentor. The senior graduate students regularly work with newer students explaining concepts and sharing old notes, helping with homework assignments, and providing study guides for examinations.

Lesson 5: Some Maintenance Is Needed

Some maintenance of support mechanisms is needed. Though the establishment of a community of Black mathematics graduate students has been very important in the recruitment and retention of many Black graduate students, the community alone does not appear to be sufficient to achieve success in developing more Black Ph.D. mathematicians.

Of the 14 Black students who entered one of the mathematics Ph.D. programs in 1993 and 1994, six have advanced to candidacy and are thus very likely to complete the degree requirements. While that is a little less than half who should succeed in earning the doctorate, the fraction compares favorably to figures for attrition both for Black students nationwide and for the general mathematics graduate student population at Maryland.

However, the results are not as satisfactory from the next two entering classes. Of the seven Black students who entered the graduate program in 1995 and 1996, only one has advanced to candidacy. Two others who departed in 1999 are keeping the door open to the possibility of returning to continue for the Ph.D., but there is great uncertainty. The difficulties of the latter seven students seem to have coincided with a decline in *direct* faculty support and community-building activities. Faculty are in a position to give a broader perspective on matters versus a graduate student perspective, which is often governed more by expedience than by experience. Senior graduate student leadership, while valuable, is no substitute for direct faculty guidance, especially for new (i.e., first- and second-year) students.

Despite some difficulties, the numbers are encouraging enough to suggest that something good is happening here. If all six of the aforementioned Ph.D. candidates do receive the degree in the near future, then we at Maryland will match the typical annual *national* numbers for African American mathematics Ph.D.s from just two entering classes of students. ■

Notes on Enlarging the Pool of Black Math PhDs:

Lessons at the University of Maryland

by Duane A. Cooper, University of Maryland

As of late 1998, the University of Maryland's Department of Mathematics boasts 21 Black graduate students, 17 of whom are pursuing the Ph.D. with 9 advanced to candidacy. This group of 21 is widely believed to be the largest concentration of Black mathematics graduate students at one institution in the U.S. Its size is significant because, in a typical year, only a half-dozen African Americans earn a mathematics Ph.D. During the 1990s as this population has arisen, we at Maryland have been learning many lessons about the recruitment and retention of Black mathematics graduate students. The lessons highlighted here are the subject of an extensive examination of our graduate students' experiences entitled "Changing the Faces of Mathematics Ph.D.s: What We Are Learning at the University of Maryland", a chapter of *Changing the Faces of Mathematics, Volume 3: Perspectives on African Americans,* an NCTM 1999 publication.

Lesson 1: One Individual Can Make a Difference

One individual can make a tremendous difference, though departmental commitment is recommended. Raymond L. Johnson, for many years the department's only Black professor, was instrumental in bringing forth Maryland's large Black mathematics community. It was during Johnson's five-year tenure (1991-96) as department chair that the large cluster of Black graduate students arose.

Johnson denies credit for bringing in the students, emphasizing that the department made the decisions on whom to admit. Several students, though, cited Dr. Johnson as a reason they chose Maryland for their doctoral study having met him at recruitment events such as Undergraduate MATHFest, an annual conference sponsored by the National Association of Mathematicians designed to inform Black mathematics majors about and inspire them to consider graduate and career opportunities in mathematics.

Early in his chairmanship, Johnson was struck to see that three Black graduate students who had entered the program together barely knew one another. He began working proactively to nurture a sense of community among the Black graduate students, and as chairman, was able to provide departmental support to sponsor initial social gatherings. Today, the Black graduate students continue organizing gatherings for special occasions like celebrations of achievements of program milestones, but also just to get together. Over several years, Johnson met regularly with the Black students in an effort to develop the Black student community as an information network about practices and people.

Lesson 2: A Black Presence Can Attract Students

A Black presence on a department's faculty, staff, and student community can attract students to matriculate. Recruited students have remarked about the extensive Black presence beyond just the student population. They noted the racial diversity of the staff- in contrast to what they observed on their campus visits to other institutions- which made them feel comfortable in the department. And while there are only two African Americans among the fairly large

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