Mathematicians and
Education Reform
...Forum...

Summer 2001 **Special Issue** Volume 13 Number 3

Mathematics Immersion Experiences in Professional Teacher Development Programs

by Al Cuoco and Nancy Antonellis, Education Development Center, Inc. (EDC)

It has been generally recognized for a long time that there exists a pressing need for a happy union of two extremes in the training of mathematics teachers—the one extreme emphasizing the methods of teaching to the exclusion of the consideration of the mathematical content, and the other extreme allowing an early preoccupation with the highly technical detail of some specialized mathematical subject with the consequent neglect of the broader mathematical culture so essential for the effective teaching of mathematics.
—Arnold Ross (1963)

Effective professional development for practicing mathematics teachers comes in many forms. One taxonomy, outlined in [1], gives several possibilities:

• One form of professional development involves studying *curriculum*—the effective implementation of a curriculum, curriculum development, or curriculum adaptation.

• Teachers may also study "case discussions" or examine student work to develop awareness of student thinking.

• Forming study groups that meet regularly to discuss curriculum, mathematics content, or implementation is one effective way for teachers to structure their own professional development.

Other models of teacher professional development include:

• Studying mathematics "at the site of practice"—professional development programs based on the mathematics you teach.

• "Profound Understanding of Fundamental Mathematics" [2]—studying your own curriculum materials in order to better understand the mathematics you teach.

• Task design—studying the mathematical and psychological principles of designing student activities.

• Japanese Lesson Study (jugyou kenkyuu)—working in collaborative teams to carefully design, test, and refine lessons on a particular topic.

continued on page 13

Inside

Featured Projects

Announcement

Building Teacher Leadership in Mathematics Institute
February 8-9, 2002
Cambridge, MA

EDC invites you to attend a two-day institute on February 8-9, 2002 to learn about the design and implementation of Building Regional Capacity. The ***Building Teacher Leadership in Mathematics Institute*** will take place in Cambridge, Massachusetts. Funding for the two-day institute is provided by a grant from the NSF. Participants' lodging and some meals are covered by the grant, and some travel stipends are available. The number of participants will be limited to about 50 people.

The ***Building Teacher Leadership in Mathematics Institute*** is designed to bring together present and future leaders from around the country in order to:

- Address the challenges of improving the teaching and learning of mathematics in our middle and high schools
- Help participants think differently about how mathematics professional development programs are designed
- Describe a successful mathematics professional development program, and discuss ways to adapt it to meet your local needs.

For more information and to receive registration materials please contact:
BRC Conference Coordinator, Education Development Center, Inc., 55 Chapel Street, Newton, MA 02458; brc@edc.org; (617) 618-2431.

Contact Information for Immersion in Mathematics Projects for Teachers

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University of New Hampshire Master of Science for Teachers: A Mathematics Degree for Mathematics Teachers

by Steven R. Benson, Center for Mathematics Education, Education Development Center

Background and Overview

The Master of Science for Teachers (MST) program at the University of New Hampshire began about 40 years ago as an NSF Summer Institute hosted by the UNH Mathematics Department. In the 1970s, when the NSF stopped funding such institutes, the program became self-supporting. The MST is a three-summer residential program, although some teachers commute from the local area. The summer sessions are 7 weeks long. Each year, between 50 and 65 teachers are enrolled in the program, with about the same number of male and female teachers. The majority of our participants come from the Northeast US (most from New England), but we have had students from just about every state, including Hawaii and Alaska, as well as from Canada, Turkey, Saudi Arabia, Nigeria, and Brazil.

In addition to their geographical diversity, MST teachers come with a wide variety of educational backgrounds and teaching experience. The vast majority of teachers who are admitted are high school teachers. Although it is relatively unusual for middle and elementary school teachers to have the prerequisite mathematical experience to enroll in our program (the equivalent of a mathematics minor), some middle school teachers have successfully completed the MST program. We continue to explore the feasibility of developing options specifically targeted toward this population of teachers.

What attracts teachers to the MST program? In a recent survey of current and past MST teachers most of the teachers replied that they were interested in pursuing an advanced degree, but wanted to learn more mathematics (many respondents expressed frustration and disappointment in the education courses they took as undergraduates). One teacher summed up the opinions of many: “I had been looking for a masters program for several years. I did not want to enroll into a program that focused on math curriculum or teaching methods. I felt my credential program (at the graduate level) and summer workshops I had attended covered these areas sufficiently. I wanted to ‘beef up’ my mathematics background. I read the ad for MST in the *Mathematics Teacher* and applied immediately.”

Many teachers also appreciated the fact that the MST is a summer program, which allows them to focus on their own studies rather than having to split their attention between study and teaching as they would have to do in an academic year program. In addition to the mathematics, an important aspect of our program (as reported by the teachers) is the camaraderie engendered by being thrust together in such an intense situation. Every single teacher mentioned the friendships he or she had made, when asked what was the most important nonacademic aspect of the MST. Many expressed hope that, through e-mail correspondence, they could keep these relationships intact after graduation:

“Meeting and making the close and meaningful friendships at MST is something I never expected. Thank goodness email is available to most of us. With the time difference, it’s practically the only means of communication for me. I will miss not heading off to the East Coast this summer because it means I will not be living with [friends in the program].”

Except for the years during which the MST was an NSF Institute, no financial aid, other than student loans, has been available to our participants. However, many are at least partially reimbursed by their schools or districts. Furthermore, we are proud to announce that 2001 is the initial year of funding the James R. C. Leitzel scholarship by the UNH Foundation. Each year, an MST student will be chosen to receive the award, given in memory of our friend and colleague, Jim Leitzel, who was involved deeply in improving mathematics education, particularly teacher education.

As curriculum goes, much of our program is similar to other content-based mathematics programs for teachers. While the majority of our courses focus on content, instructors use a variety of approaches and pedagogical styles and seek to build connections between the mathematics they are teaching and the mathematics the participants teach. The program, which requires teachers to complete 30 units of course work, is typically completed in three summers. Eighteen of the units required for graduation must come from the “core” of 2 Geometry, 2 Algebra, and 2 Analysis courses, which are offered each Summer. In addition, new participants often complete a transition course, titled *Mathematical Proof and Problem Solving,* which typically includes a review of proof techniques, basic Logic and Set Theory, and Mathematical Problem Solving. This course has been in flux the last several years as it has evolved from a predominantly symbolic logic and set theory course to one which uses problem solving experiences as avenues to advanced mathematics and proof. We also regularly offer 3-unit elective courses (recent offerings include Number Theory, Topology, Discrete Mathematics, and Technology in Teaching Mathematics), and 1-unit courses on topics in mathematics education (topics in 2001 were Assessment, Mathematical Software, and Teaching Strategies). Based on this model, we are discussing the possibility of offering 1-unit content courses that go beyond the core sequence by choosing more advanced topics or covering a specific history of the topic area. Most 3-unit courses meet 2 1/4 hours per day, 5 days per week, for 3 1/2 weeks, and the 1-unit courses typically meet 2 1/4 hours a day, 3 days a week, for 2 weeks. *Mathematical Proof and Problem Solving* is offered 2 hours per day, 3 days a week, for 7 weeks.

Social Features

This is a high intensity program due to the compressed time-frame, but the teachers are up to the challenge. As a recent graduate put it: "I love that the program doesn't seem to sacrifice high expectations just because we meet for a condensed period of time.'' Having the teachers live close to campus (either commuting every day or living on campus) leads to a strong sense of camaraderie among the MST participants and staff. In the dorm, there's nearly always a group of students discussing homework or an upcoming exam. One student put it this way: "My favorite *academic* part of the program is the intense nightly homework sessions in the dorm. It was nice to walk down the hall and check with others after working 4 hours on my own.''

It's clear that the teachers, being willing and able to put up with an intense class schedule, are motivated and dedicated to learning mathematics, but they also take some time (however limited) to have fun every now and then. We typically have barbecues at local parks and there have been occasional excursions to the White Mountains, a winery, and a brewery in past summers. Our softball team, *Cauchy's Dogs*, is an enjoyable diversion for a number of the teachers. While we've never won the Summer League Championship, fun is had by all. *One-to-One*, our weekly newsletter, includes entertaining accounts of the softball game, announcements for upcoming excursions, and a problem of the week (submissions of correct solutions wins a coupon for ice cream at the UNH Dairy Bar). While free time is a premium, MSTers know how to spend it when they have it.

The *New* Concluding Experience

Until recently, the final requirement for graduation was passing a series of comprehensive examinations in the core content areas. Starting in the summer of 1997, however, we introduced the new concluding experience, which was made an official requirement of the program for those who have enrolled in the program since that summer. The concluding experience is an assessment tool that is fundamentally different from the comprehensive examination format, and the staff and teachers alike have found it to be a successful and appropriate alternative to the exams. As part of the concluding experience, graduates are required to complete a mathematical portfolio and to successfully complete a comprehensive problem set and associated *Problem Solving Seminar*. Since this is a unique aspect of our program, we will now take some time to describe it in detail.

Ideally, the *mathematical portfolio* should represent a chronicle of the teacher's mathematical career during the time he/she was enrolled in the MST program. A brief summary of the Portfolio requirements follows (for more information on creating and assessing mathematical portfolios, see [2] and [3]):

MST Portfolio requirements

- A letter to the reader outlining the author's vision and goals for the MST program and how the pieces included reflect these goals.
- A presentation, talk, workshop, or article, developed in consultation with a faculty advisor, that brings together several mathematical ideas.
- Responses to a set of questions that pre-college students might ask.
- A description and an explanation of the mathematical insight that most excited the participant over his/her mathematical career.
- A 1-2 page review of 2 articles, one in mathematics content at the level of *the American Mathematical Monthly* and one in mathematics teaching at the level of the *Mathematics Teacher*.
- A collection of 'best work.'
- A mathematical autobiography describing the teacher's development as a learner of mathematics.
- An annotated list of courses taken.

Some of the above items are fairly common components to a portfolio, but some of these are unique, so we discuss them here. The required responses to a set of questions that a precollege students might ask were motivated by *Mathematical Questions from the Classroom* by Crouse and Sloyer, [1]. In addition to addressing the student's question in detail (perhaps imagining a dialogue with the student), participants must also discuss how their MST experience has helped prepare them to answer such questions. Typically, teachers are required to answer two of three or four questions. A sample of two *Questions a Precollege Student Might Ask* that have been used in the last few years are:

- "Isn't .999999999… really a tiny bit *less* than 1?'' Please provide 3 separate responses to the question, depending on the "audience'': (1) a 5th grader; (2) a student in grade 8-12; (3) another MST teacher or instructor (in this case, a careful mathematical argument is preferred, rather than an intuitive rationalization, which may or may not be appropriate for the other audiences).

- A former student, now a mathematics major at *Your State U*, has come to ask you a question she's uncomfortable asking her professor. "I'm taking *Math ABC: Introduction to Modern Algebra* this semester. I'm doing OK, so far, but this stuff sure doesn't look *anything* like Algebra to me! What does Modern Algebra have to do with *real* Algebra?''
  In addition to being curious, she's a little concerned that she's missing the BIG PICTURE. Address her concerns, using your own Algebra experiences as a guide. What are some of the connections between Abstract Algebra and High School Algebra? There must be some— otherwise we wouldn't call it Algebra, right?

By thinking about mathematical questions that could come from one of their students, we believe MST participants are building a stronger pedagogical content knowledge base than would come from merely discussing mathematical concepts in the context of their own classes.

The mathematical insight description and the article review requirement have caused some students difficulty. In each of these components, as well as the "questions from the classroom'' requirement, students sometimes struggle to address the questions with sufficient mathematical and/or pedagogical depth. We have learned through the years to be more specific in our instructions and to provide timely feedback, which has helped a lot. We highly recommend that teachers submit drafts of these sections early in the summer so that we can provide timely feedback well ahead of the September 1 deadline for December graduation.

A piece of the portfolio that has been especially successful is the collection of best work. In this section, participants are given the chance to individualize their work, focusing on what they perceive to have been their strengths in the 3 years they've spent in the program. Often, teachers include one or more entries from the Problem Set (see below), since they've spent so much time on it during their final summer.

The second half of the concluding experience is the Problem Set. The problem set consists of 15-20 mathematical problems and is completed during a participant's third (final) summer in conjunction with the Problem Solving Seminar. The majority of the problems involve one or more of the core topic areas, since one of the goals of the problem set is to encourage teachers to synthesize and apply the mathematics they've learned in the last 3 summers. In addition, some problems are chosen from non-core areas, such as Number Theory and Polynomials. The Problem Solving Seminar includes a study of a variety of problem solving techniques, a review of content from the core sequence, and some new concepts, as well. Although specific problems from the Problem Set are never discussed during the seminar, some topics relevant to the problem set are included. Some of the topics "covered'' in the seminar are problem solving heuristics, inequality theorems, basic theory of equations, the pigeonhole principle and other counting theorems, modular arithmetic techniques, and a review of group theory, linear algebra, limits and sums of sequences, and various Euclidean and non-Euclidean geometry results. Loren Larson's text *Problem Solving through Problems* [4] has been an excellent resource for the class for both the teachers and the instructor. Listed below are some samples from recent problem sets.

- On each side of an arbitrary triangle, construct the equilateral triangle that lies outside the original triangle. Under what circumstances will there exist a point X which is common to all three circumcircles (of the three equilateral triangles)? Did you make any Euclidean assumptions? If so, does the Theorem still hold true in neutral geometry (geometry that is based on only the first four of Euclid's postulates, i.e., is neutral on the parallel postulate)? What about hyperbolic or elliptic geometry?

- Let G be an Abelian group and define H to be the set $\{g^2 \mid g \in G\}$. What can be said about the quotient $|G|/|H|$? Be specific — list all possible values for $|G|/|H|$.

- Let n be a positive integer and define $s(n)$ to be the sum of the digits of 2^n. Find all n so that $s(n)=s(n+1)$.

• Does there exist a curve $y = ax^3+bx^2+cx+d$ (with $a\neq 0$) and a line l so that l is tangent to the curve at *two* distinct points?

• Show that the circle inscribed within a Pythagorean triangle (the lengths of the sides are integer-valued) has an integer-valued radius.

Teachers are required to complete at least 10 of the problems from the set and must "defend'' their problem set before a faculty committee. This hour-long defense typically involves the teacher making an oral presentation of his or her results to the committee. Although all teachers so far have passed their defense, it's also the case that all but a small handful have also been required to make (sometimes substantial) changes to their problem set before they are "cleared'' for graduation. As with components of the portfolio, teachers are encouraged to submit drafts of solutions during the summer to avoid surprises at the last minute. Encouraging draft submissions has proved to be very successful and has resulted in higher quality portfolios and problem set solutions and less major rewriting required at the end of the summer.

Everyone involved in the program has been extremely satisfied with the new Concluding Experience and have found it to be a more authentic, satisfying way to assess and reflect on three years of participants' hard work and mathematical growth than the comprehensive exam requirement. In our opinion, preparing the portfolio and problem set solutions is much more pedagogically sound than cramming for a series of two-hour exams. Participants feel a sense of pride and accomplishment when they turn in their portfolios and problem sets. In contrast, the experience of taking the comprehensive exams was first dread in anticipating the exam, and then relief in finishing the exam. Every now and then we'll receive emails from graduates who either send us (or ask for) a solution to a problem they couldn't solve during the previous summer, or tell us of one of their students who asked them a question similar to one of the "questions from the classroom.'' That rarely happened when we required comprehensive examinations in the core areas for graduation. In most cases, when the exam was finished, the teachers seemed to stop thinking about the mathematics.

In the Teachers' Words

While it's certainly appropriate to conclude our discussion with the above description of the MST concluding experience, we have decided to give some recent graduates the last word. Here are some responses to the question, *Would you recommend the MST program to your friends and colleagues?*

"Though the program brought challenges I never dreamed of experiencing, both intellectually and emotionally, … I am proud to say I am a graduate of the MST program."

"The MST experience made me appreciate the rigors of being a student again!! Secondly, this program allowed me to deepen my understanding of mathematics and take it back to my classroom."

"I'd recommend MST for anyone who is serious about being a better math teacher. It's not for someone who just wants to move up the pay scale."

"If a colleague was looking for an easy way to get a degree I'd steer them somewhere else. Anyone who might be interested in a program that will stretch them a little and encourage them to think about math in new ways has already heard me sing MST's praises."

"I have already recommended MST to friends and colleagues that are serious about building a solid mathematics foundation. I caution those that are only interested in earning a higher degree for a salary increase. MST is *not* for them."

"I would recommend this program without reservation. I would be very upfront about the stress induced by moving through material at such a rapid pace, but it was well worth it. The friends that were made in this program will be so for the rest of my life."

References

1. Crouse, R. J. & Sloyer, C. W. (1987). *Mathematical Questions from the Classroom*. Dedham, MA: Janson.
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3. Karp, K. S. & Huinker, D. (1997). Portfolios as Agents of Change. *Teaching Children Mathematics*, 3(5), 224-28.
4. Larson, L.C. (1983). *Problem Solving Through Problems*. New York: Springer-Verlag. ■

PROMYS for Teachers: Learning in the Spirit of Exploration

by Glenn Stevens, Boston University

PROMYS for TEACHERS is a collaboration of Boston University's Department of Mathematics with the Education Development Center in Newton, Massachusetts. Together we are working to promote what we call a "culture of exploration" in high school mathematics classrooms. Our experience proves that the joys of exploration and discovery can be experienced by high school students and teachers in ways that are not all that different from what a seasoned mathematics researcher experiences. Moreover, teachers who have such experiences in their backgrounds are better prepared to encourage independent inquiry among their own students. Reports from former PROMYS teachers confirm that *all* students benefit from this kind of instruction, not just the most talented ones.

History of PROMYS

The Program in Mathematics for Young Scientists (PROMYS) has existed for over a decade at Boston University as a program that engages mathematically inclined high school students in the process of mathematical exploration through their work on unusually challenging problems in number theory. Since 1991, PROMYS has also worked with pre-service high school mathematics teachers from the Boston University School of Education. Beginning in the summer of 1999, PROMYS expanded its activities again by engaging in-service Massachusetts high school mathematics teachers in the program's summer activities and running five professional development seminars at the Education Development Center (EDC) during the academic year. In the last two years, PROMYS for Teachers has received funding from NSF, Massachusetts Eisenhower Professional Development Program, Toyota USA Foundation, MathWorks, NEC Foundation of America, and Park City Mathematics Institute.

PROMYS has its roots in the famous Ross Young Scholars Program, which is still running strongly at the Ohio State University in Columbus forty four years after its beginnings at Notre Dame in 1957. Many of the PROMYS faculty are alumni of the Ross Program and much of our inspiration comes from the experiences we shared in the late sixties and early seventies as young participants in Arnold Ross's program.

Elements of the Program

PROMYS for TEACHERS is built around three major components: (1) immersion in an experience of mathematical exploration for six weeks in the first summer; (2) reflection on the relevance of this experience for classroom practice during the intervening academic year; and (3) more immersion in mathematics in the second summer. Teacher participants receive a stipend of $1,800 for each summer of participation in PROMYS. In addition they receive 8 graduate credits in mathematics for their work in the summer and an additional 4 graduate credits in mathematics education for their work in the academic year workshops.

The PROMYS Community

The PROMYS community is divided into several closely interacting groups who bring a variety of experiences at a variety of levels to the program. The first-year participants consist of 10 in-service teachers, 5 pre-service teachers, and 45 high school students. Another 10 teachers attend the program as returning participants along with 15 returning high school students. Teachers commute from home each day, but the students, who are recruited nationally, live in the dormitories. A staff of 8 counselors works with the teachers on problem solving activities during the day and reviews their written work each evening to be returned with comments the next morning. The teacher counselor staff is made up of former PROMYS teacher participants and graduate students from the department of mathematics. An additional staff of 15 undergraduate counselors lives and works with the high school students in the dormitories. The student counselors are recruited from around the country. Many are former participants in the program. The PROMYS faculty has 7 members, two of whom come from the Education Development Center while the other five are university mathematicians, many of whom were participants in the Ross Program in their high school years. The faculty work together with participants on problem solving and in exploration labs.

The significance of the depth of this community cannot be overemphasized. PROMYS aims to change participants' attitudes about mathematics. The community is by far the most effective tool for bringing about this change. Most participants find their experience in PROMYS to be overwhelming at first. They must work hard to adjust to the demands of PROMYS and its unusual philosophy of learning. Beginning participants work together in groups and often discuss their ideas with returning participants and counselors. Returning participants share their valuable insights and offer encouragement to the beginning participants. Assuring that the participants find the challenges of PROMYS to be exciting and fulfilling is probably the most important function of the counselors. The role models provided by returning participants and counselors, and the personal and accessible feedback they offer, are indispensable features of the PROMYS experience.

"The community is by far the most effective tool for bringing about [the change in participants' attitudes about mathematics.]"

The Summer Program

The PROMYS summer activities combine three tightly interlocking components: (1) problem-solving activities shared by all first-year participants; (2) independent team projects carried out by all participants in groups of four; and (3) advanced activities (seminars, and projects) for returning participants. These components are supported by discussion meetings and independent problem-solving activities with the counselor staff.

(1). Problem Solving Activities. All first-year participants engage in intense problem solving in number theory designed to introduce them to the methodology of mathematical research. We use the magnificent collection of problems designed by Professor Arnold Ross for use in the Ross Program. These problem sets provide participants with valuable insights into the research process. Particularly effective is the way the problem sets build on themselves over the course of the program. Simple numerical observations made in the early problem sets are enriched and extended in later sets so that participants are treated to a first-hand experience of unraveling deep and significant truths out of simple ideas. Lab projects (described below) build on particular threads that weave through the problem sets and introduce participants to some of the many unsolved problems that still remain. By engaging all participants in shared problem-solving activities, PROMYS creates a strong community of closely interacting teams working on independent but related projects, thus recreating the crucial element of communication and sharing of ideas that is so important to all research mathematicians.

(2). Team Activities. Every participant is assigned to a team of four participants who will work together, in consultation with the PROMYS faculty and staff, on an independent open-ended project. These projects extend particular facets of the general problem-solving activities described above. Each team consists of one teacher and three students, sometimes including a student from the teacher's high school. The topic for the project is selected from a list of proposals prepared by the PROMYS faculty. Each project contains parts that are immediately accessible to all participants, and then continue with open-ended explorations, including unsolved problems. The summer 2001 projects for first-year participants include explorations of the postage stamp problem, generating functions, the calculus of finite differences and sums, partitions, the game of SET, repeating decimals in various bases, congruence properties of Pascal's Triangle, Pick's formula and Euler's Theorem on simple graphs, Egyptian fractions, and sums of squares.

Each team member keeps a notebook for recording progress as it is made, as well as new ideas, conjectures, and questions that arise along the way. The results of numerical experiments are shared and preserved in these notebooks. At the beginning of the third week, team progress reports are reviewed by the PROMYS counselors and faculty and are returned with feedback and suggestions. In the fourth week, the results of each project are written up carefully in the form of a mathematical research paper and submitted to the PROMYS faculty and in the fifth week each team jointly gives an oral presentation of their work to the rest of the program.

(3). Activities for Returning Participants. Returning participants take part in advanced seminars designed especially for them on topics that vary from summer to summer (e.g. discrete mathematics, geometry and topology, and the theory of equations). The PROMYS 2001 offerings for returning participants are: Combinatorics; Random Walks on Groups; and Analysis of Algorithms. Like the first-year participants, the advanced participants are also divided into teams of four, including one advanced teacher participant, and these advanced teams work together on projects designed to connect quickly with accessible research projects, the results of which may be publishable in research journals.

The advanced projects in 2001 include investigations of arithmetic functions in number fields, generalizations of perfect numbers, change-making problems in combinatorics, and decompositions of the circle.

Returning participants add an important dimension to PROMYS by sharing their valuable program experience and by serving as peer role models for first-time participants. The second summer of participation helps teachers cement their experience of mathematical exploration and extend their content knowledge into other mathematical fields relevant to the high school curriculum. Moreover, through their discussions with beginning participants, the returning teachers share strategies they have learned for engaging students in the process of mathematical exploration. This is an important part of the community-building that is so crucial for the long-term success of these efforts.

Academic Year Pedagogy Workshops

The workshops are designed and delivered by Al Cuoco and Michelle Manes of the Education Development Center (EDC) with support from Boston University's Department of Mathematics and School of Education. Teachers meet with mathematics education researchers, mathematicians, and other teachers to discuss curriculum issues, design projects, and to develop teaching methods that will engage the broadest possible spectrum of high school students in meaningful mathematical exploration. The seminars help teachers develop age-appropriate research experiences for their entire spectrum of students and to "unpack'' the pedagogical approaches used in PROMYS to enrich the school curricula. Another important goal of the workshops is to establish an ongoing network (including an electronic network) of teachers, mathematics educators, and research mathematicians.

Each workshop begins with a concrete mathematical activity for the teachers that is easily accessible to students. Then deeper aspects of the activity are examined, in some cases leading right into contemporary research articles. Past topics have included cryptography, complex numbers and chaos, Pythagorean and Eisenstein triples, and areas (and volumes!) through paper-cutting and folding. Each workshop also provides time in which individual teachers can present samples of their own classroom activities and to discuss actual situations that have arisen in their classrooms. In the afternoons, teachers may design curriculum units for use in their classrooms.

Drawing on years of experience designing excellent curricula for the high schools, Al and Michelle are able to give the teachers valuable insights into the preparation of classroom activities centered in exploration. At one workshop, the participants were treated to a view of five or six different textbooks in geometry, all in current use. The discussion centered on an analysis of each, in order to understand how each was designed to accomplish its own distinctive goals. Another workshop was devoted to ways in which Geometer's Sketchpad can be used to stimulate student explorations. Other workshops explored Computer Algebra Systems and using the internet as a research tool.

One of the most gratifying features of the workshops has been the ongoing participation of former PROMYS participants. PROMYS teachers are expected to participate in only one year of the academic year workshops, but in fact almost every teacher from the first year attended at least one workshop in the second year as well, and many attended several. This attests to the high value that teachers place on the EDC workshops and is also an indication of the coherence the teachers feel as a group. The closeness of this mathematically-centered community is undoubtedly the most valuable asset of PROMYS for Teachers.

PROMYS Goals and Philosophy: A Culture of Exploration

PROMYS for TEACHERS is designed to support current efforts nationwide to *change the culture* in high school mathematics classrooms to one centered on exploration and problem solving. The program fosters a *community* of teachers, young people, research mathematicians and educators all working together and sharing intellectual challenges. The program activities are designed to encourage the kinds of *habits of thought* that every researcher (not just of mathematics) uses in his or her efforts to understand the world around us. This combination of community and rich intellectual environment is a potent tool for engaging participants at a very personal level in an experience of exploring significant ideas.

Exploration is at the heart of mathematics. Mathematics is not just a powerful tool for *understanding* the world around us; it is also a powerful tool for *discovering* the world around us. When students learn mathematics through exploration, they experience a side of our subject that lies much deeper than the simple skills of algebraic manipulation and calculation. There are many opportunities in the high school curriculum for students to experience mathematical discovery. Teachers need the preparation and the confidence to recognize and take advantage of these opportunities. Here are some reasons for including a strong component of immersion and exploration in the professional development of teachers.

To emphasize the human side of mathematics. Intellectual exploration is unique to the human experience. For students, mathematics often appears to have a rigid and hostile quality, which feels very different from the spirit of other human activities. By exclusively emphasizing calculation and symbol manipulation in the classroom we reinforce these misconceptions. Exploration, on the other hand, can be done in many ways. Teachers find numerous opportunities in PROMYS to try out some of these ways for themselves. The adventures they experience come complete with the frustration of dead ends, the excitement of finding a promising new trail, the sense of triumph at surmounting a hard problem, and the sense of wonder at the beauty of the unexpected mathematical vistas that come into view. Teachers who have had such an experience of learning in the spirit of exploration are better prepared to bring this spirit to their classrooms. One PROMYS teacher told us: *… the passion for and the approach towards mathematics can only be learned by being immersed in mathematics.*

Another teacher wrote: *PROMYS has had profound effect on my teaching, mostly by reminding me why I love mathematics in the first place—the joy of discovery.*

To encourage student independence. Learning is an intensely personal experience. No two students understand mathematics in the same way. Students rarely learn by listening to teachers lecture in the classroom. They learn from experiences that engage them personally and at their own level. By emphasizing exploration in the classroom, teachers can foster a climate in which such experiences occur frequently. One teacher described this as follows: *I'm now having great success with students of all ability levels. Posing the proper questions allows my brighter students to dive deeper into topics than their classmates. More importantly, my slower students feel good about making progress at their own pace, resulting in increased confidence (perhaps the best motivator).*

To prepare students for life in a rapidly changing world. The importance of flexible learning skills is especially apparent in these times of rapid technological progess. To keep up with international economic competition, it is increasingly apparent that the American workforce needs learning skills that adjust easily to changing demands. An experience of immersion in mathematical exploration can give teachers added confidence in their own abilities, so that they can recognize and appreciate unorthodox student approaches to problems and respond with the kind of flexibility that encourages independent student thinking. Here is how one teacher describes this process: *By viewing my role as a "guide to student learning" rather than an instructor, I can remain focused on asking the right leading questions and resisting the temptation to show them what I know.*

To develop habits of thought that favor hard problems. It is all too common that students tremble with trepidation in the face of hard problems. By emphasizing the process of discovering mathematics along with the results of those discoveries we can foster a student culture which relishes the prospect of exploring the unknown. Of course, this also requires readiness on our part to reward students for unsuccessful attempts. As one teacher expressed it: *I find that as a teacher it is sometimes easy to forget that our students don't always understand everything we teach …. One thing that PROMYS helped me to do is to help students to learn to appreciate being challenged and frustrated because that means they are thinking about the problem. PROMYS helped me to become better at asking questions rather than just giving answers.*

To encourage sharing of ideas. Everyone enjoys talking about interesting discoveries they make. Such interaction can be organized formally in the classroom by asking students to work in small groups, or by asking students to describe their ideas to the class. Best of all, however, is when students independently talk to each other outside of the classroom. By offering encouragement and support to teachers as they use open-ended problem-solving techniques in their classrooms, PROMYS hopes to contribute to a changing culture in the schools that will inspire many interactions of this kind.

Exploration is what mathematicians really do. Learning through exploration provides students with realistic opportunities to experience mathematics the way a professional scientist does. It is impossible to understand the process of mathematical research without having experienced it for oneself at some level. Exploration in the classroom provides opportunities to discuss the dynamic nature of modern mathematics in a context that simulates actual experiences of mathematical discovery. To lead these explorations effectively, teachers must feel and think like mathematicians themselves. One PROMYS teacher summed up his approach to teaching as follows: *These are not things that I do consciously. Rather, they're merely reflections of how I have learned to approach mathematics. Thus, I could've only learned how to teach the way I teach by studying mathematics intensely on my own.* ■

Building Regional Capacity: A Collaborative Approach to Professional Development Through Problem Solving

by Kenneth Lavasseur, University of Massachusetts Lowell

Introduction

Have you ever spent an extended period of time trying to do mathematics alone? Most of us in higher education are fortunate not to be in that situation, but the typical K-12 teacher who wants to do some mathematics tends to be isolated. There are other math teachers in most schools, but they just don't do mathematics spontaneously — at least not in most schools here in the US. The program that I will describe aims to help change this situation. Put into a broader context, the program's goal is for the participants to increase their own mathematical activity and that of their colleagues *in order to support learning. Building Regional Capacity* (BRC), a collaboration of Education Development Center (EDC) and the University of Massachusetts Lowell (UML), is a professional development program for mathematics department heads and lead teachers in grades 6-12. BRC participants attend two consecutive summer institutes at the UML where they engage in graduate level mathematics courses and learn to deliver effective professional development programs to mathematics teachers in their own districts. Central to the program was participant reflection on how they, their students, and their colleagues approach mathematics. There have been two cohorts in BRC. The first cohort of 17 participants started this two-year program in 1999 and a second cohort of 46 participants started in 2000.

The Summer Institutes

In a series of two-week summer institutes at UMass Lowell, participants have been immersed in mathematics. Initially, we were faced with the challenge of convincing them that this was something that would be beneficial in their own teaching; and that this could also be made to appeal to their colleagues. Roughly two thirds of the first year institute's 80 hours were devoted to a course in mathematical problem solving that is described below. The remaining time was spent on professional development issues. Significant time was spent discussing how the study of student work on problems that they and their colleagues work on can provide a deep understanding of student learning. This was a major selling point for the mathematics immersion. Participants became aware that the kinds of *habits of mind* (see page 13) that they needed in their problem solving course were precisely the ones that their student needed for a wide range of problems, including the open-ended questions that were being developed in state assessment tests.

Kick-off Day

One major design change in the institutes between years one and two was the Kick-off, a full day of introduction to the program. Since we were cramming the equivalent 1 1/3 graduate courses into two weeks we needed to use our time as efficiently as possible. After the first year with a relatively small cohort, we realized that the time for personal introductions, general introductions to the program, etc. could be done more effectively in a separate day in the spring. This also let us assign readings and to give the participants in the problem solving course "baseline problems" to work on in the following weeks. The participants' responses gave the instructors a feel for where they stood mathematically and also established expectations of what kinds of problems they would be doing in the summer. A particularly memorable part of the Kick-off was a presentation by some participants in Cohort 1 describing their own experiences and emotions, running from fear to enjoyment, in their previous summer's institute.

The Problem Solving Course

The course that all participants in the institute took in their first institute was a graduate course titled Mathematical Problem Solving, which is part of the Master's program for mathematics teachers in the UML Department of Mathematical Sciences. Some of the underlying theory of problem solving, as discussed by writers such as Polya and Schoenfeld, is touched on in the course; but most of the course is devoted to *doing mathematics*. The primary objective of the course within the structure of BRC was to convince the participants that developing useful habits of mind should be central to their teaching of mathematics.

Outside the classroom, each participant worked on a problem of his/her choice from a recent MAA journal.

These problems from *The American Mathematical Monthly, Mathematics Magazine, The College Math Journal*, and *Math Horizons* are quite difficult. It was made absolutely clear to everyone that finding a complete solution to the selected problem would be a major accomplishment for them and that any kind of partial success would be highly valued. Partial success could include, among other things:

• Solution of a simpler or related problem;
• Identification of alternate representations of the problem;
• Visualizations of the problem.

The MAA problems served as a “laboratory” for applying the principles that were discussed and applied in the classroom to short-term problems. The participants selected the problems to work on so that they were comfortable with the mathematics involved, at least on the surface level. Participants were allowed to work together if they selected the same problem as long as they documented work separately. Some participants actively wanted to do this, while others tried to select a “unique” problem. Participants kept a journal of their work and prepared a poster to summarize their progress. A poster session at the end of the institute is a culmination of their work, giving the participants a chance to share their progress on the problems.

The Discrete Mathematics Course

A discrete mathematics course in the second summer built on the problem solving course. In addition, the course served to highlight the Representation and Reasoning & Proof Process Standards in the 2001 NCTM Standards.

The Professional Development Course

A graduate education course for professional development in mathematics was completed in the course of the two summer institutes and the academic year activities. Participants learned how to organize and run in-service professional development programs in their districts, using student work and work done in the study group to explore mathematical thinking. In its second summer, each cohort completed a module from the NSF-funded *Connecting with Mathematics* professional development program (http://www2.edc.org/CWM/) and discussed implementation issues associated with these modules.

Academic Year Activities

Participants were expected to form study groups that provided 100 hours of professional development to at least three colleagues (400 teacher-hours of activity). How can we add yet another activity to a teacher’s already full plate? In most cases, we don’t. Nearly all teachers are required to participate in some sort of staff development or professional development program during the school year. The thought of actually doing some math in this time appeals to a significant number of teachers. Although the initial planning envisioned a typical study group of four teachers, a wide range of formats, sizes and settings developed. Some examples:

• One participant cooked meals for her study group. Naturally, this was very popular!

• In one district, all new teachers were asked by the school administration to attend.

• Some meetings combined study groups from different school districts.

• In some middle schools, the study groups were the first significant opportunity for mathematics teachers to meet since no “mathematics department” existed in these schools.

Several teachers experienced their first successes at leadership and fund-raising in their districts. There were some teachers who had difficulty forming a study group but some of them found success during the second year. BRC Staff conducted site visits at least once a year to meet with study groups. Depending on the needs of individual schools these ranged from recruiting visits to workshops. “Doing mathematics” was usually central to these visits. In some cases we just observed the scheduled activity and kibitzed. On other occasions we brought along some materials and tried to give the study group a “mini-institute” that included some challenging mathematics and some examination of student work. During the academic year, the program also included two full-day seminars at either EDC or UML. The agenda at these seminars was a mix of mathematics and implementation issues.

What Have We Learned?

The program has been a learning experience for everyone involved. Among the things that the staff has learned are:

• Our model seems to work. Mixing the study of mathematics with leadership skill development actually does produce a corps of teachers who are confident and capable in recruiting and leading groups within their communities. It’s an empowering mix.

continued on page 13

"Habits of mind"

The use of the term "habits of mind" was brought into discussions about designing mathematics curricula in the late 1990s by mathematics educators at EDC. We offer a brief paraphrase from their discussion to help delineate their meaning of the concept. Editor.

It is all to common to find students who do well in a particular course but cannot apply their skills outside the narrow confines of the problems presented. Are there mathematical *habits of mind* that transcend topic knowledge and allow students to apply techniques and procedures from a mathematical discipline to situations that don't explicitly look like they belong to these disciplines? It seems reasonable to look for ways of helping students develop the modes of thought that are essential for the doing of mathematics that mathematicians take for granted, but are rarely explicitly discussed in mathematics courses. Examples of the kinds of methods that can be infused and made visible throughout a curriculum are: algorithmic thinking; reasoning by continuity; combinatorial reasoning; thought experiments; proportional reasoning; reasoning about calculations; and topological thinking.

***BRC** continued from page 12*

• The process of giving participants an opportunity to share their ideas about recruiting and funding in their study groups has made us aware of the supports participants in the program need.

• Middle school and high school teachers can successfully work together on the same mathematics, although teachers at different school levels rarely have any past experience interacting. Some of the study groups also include elementary school teachers.

• We must recognize the professional development requirements in participants' districts, and assure that appropriate credit is awarded for work they do in the program. Many teachers who need professional development "points" don't have the time or energy to participate in a program like BRC unless they receive credit.

• A successful team requires bringing together people with a combination of backgrounds and skills needed for the program. Besides mathematicians you need people who have experience working with teachers.

• Teachers who have not been leaders in their systems can become leaders. We've seen significant growth among several participants who had previously not been in a leadership position.

• A pyramid effect can occur. Some of our people have 10 or more people in their groups! In at least one case, another level of the pyramid has emerged and we have study group members forming study groups. ■

A national conference on BRC, *Building Teacher Leadership in Mathematics,* will be held in February 2002 . Please see page 2 for details.

***Immersion Programs** continued from page 1*

At the 2001 Joint Mathematics Meetings in New Orleans, we organized a section of the Mathematics and Education Reform special session devoted to examples of a somewhat different genre of professional development: providing secondary teachers with an immersion experience in mathematics. This issue of the newsletter contains articles by each of the speakers in that session.

Many mathematicians have a strong sense that the most pressing professional development need for K–12 teachers is that they "learn more mathematics." But, as Arnold Ross so wisely noted close to 40 years ago, we also know that yet another course in differential equations at a local university "with the consequent neglect of the broader mathematical culture so essential for the effective teaching of mathematics" does little to enhance one's effectiveness in the classroom. The talks in this session gave three approaches to professional development centered in serious mathematics, programs that are designed to help teachers

• connect what they learn to the mathematics they teach, and

• place what they learn in the landscape of a "broader mathematical culture."

Ken Levasseur describes a program called *Building Regional Capacity* (BRC) a two year program for secondary teachers at the University of Massachusetts in Lowell. BRC consists of two 2-week summer sessions in which teachers study mathematics (problem solving and discrete mathematics) as well as effective methods for working with colleagues in their districts to help *them* learn new mathematics. To get a feel for the level of the mathematics in BRC, each participant is required to work

continued on page 14

continued from page 13

on a problem chosen from the problem sections of one of the MAA journals and to prepare a poster session describing results, progress, and conjectures. Participants found this work extremely challenging; as one teacher put it: "Putting myself in these challenging problem solving situations makes me appreciate more deeply what my kids go through."

The BRC leadership activities are also based in mathematics. Participants study various models that they can use with their colleagues, including the study of student work and the analysis of problems in their texts to uncover the underlying mathematics. BRC participants have used what they take from the program to give professional development courses of their own, to give talks at regional and national meetings, and to become mathematical resources for their districts. BRC will support a national dissemination conference in February of 2002 (for more details, see page 2).

Steve Benson and Karen Graham describe the Master of Science for Teachers (MST) program at the University of New Hampshire, which had its genesis as one of the NSF institutes of the 1960s. MST is a residential summer program featuring a strong emphasis on mathematics content, while also providing opportunities for teachers to consider alternative approaches to pedagogy. It has a core sequence of six courses (2 each in Algebra, Geometry, and Analysis), but also offers a variety of mathematics and education electives based on student and instructor interest.

One of the key features which sets it apart from many graduate programs is the "Concluding Experience" required of all graduates, for which teachers are required to develop an extensive mathematical portfolio and successfully complete a comprehensive problem set in conjunction with a problem solving seminar. In the words of one participant, "At first, I wasn't sure if MST was going to be the right program for me, because of the focus on mathematics. Over the summers spent at UNH, I found that I am really glad I chose a program with a focus on math… (T)his program has positively influenced my teaching (and) I truly understand math more than I did before MST. I feel an incredible sense of accomplishment after going through this program."

In his article, Glenn Stevens describes Boston University's "PROMYS for Teachers," an intense two-summer 6-week immersion in number theory and related mathematics at Boston University. It centers around three activities:

(1) *The courses.* Teachers take an intense six week number theory course, modeled around the Ross young scholars course, alongside high school students in the program.

(2) *The research experience.* Each teacher works with three students on a research project, similar to those used in Making Mathematics, a mentoring project to provide high school students and teachers with materials to pursue research experiences. (Further information about the research projects can be found on the web at www2.edc.org/makingmath.)

(3) *The academic year seminars.* Between the two summers, teachers attend five all-day seminars at EDC in which they work on translating the PROMYS experience into classroom practice. Teachers in PROMYS see what it's like to do real mathematics—always being at the edge of what they understand, having much more to do than they can possibly finish, and seeing hints of mysterious connections emerge almost out of nowhere. Listen to how one participant describes the experience: "*. . .* I found many beautiful connections between seemingly disparate concepts. To me, these connections are what makes math fun. So, I try to show this to my students. One example: When my 9th graders were learning how to solve quadratic equations by factoring, they quite naturally used the fact that **R** is an integral domain. But then I asked them to think about the possibility of two non-zero elements whose product is zero. And they came up with $\mathbf{Z}_6$, which we had studied three months ago. These are not things that I do consciously. Rather, they're merely reflections of how I have learned to approach mathematics. Thus, I could've only learned how to teach the way I teach by studying mathematics intensely on my own."

PROMYS graduates have developed mathematically intense professional development courses of their own—one such course was offered at the Park City Mathematics Institute this summer. Several alumni are considering graduate degrees in mathematics. The program will be featured in a series of videotapes produced by Boston's WGBH as an example of effective professional development at the secondary level.

The programs described in this issue are just three examples of the many effective programs all over the country. These programs look very different from each other, but they share an underlying philosophy: An important part of the mathematics one teaches is an understanding of what it is to *do* mathematics. These programs are at the opposite end of the spectrum from the "make and take" style of professional development in which teachers develop activities for use with their students. Their purpose is to tend to the mathematical understanding of the teachers themselves, to build on what they already know, and to help them experience first hand the processes involved in solving problems, making abstractions, and building theories.

But why? *Why should such an immersion experience in mathematics improve one's day-to-day teaching practice?* Aside from anecdotal evidence (testimonials from teachers who have gone through such programs), what are the logical arguments that lead one to believe that programs like BRC, MST, and PROMYS will help teachers be more effective at getting students to understand mathematics? Here are our candidates for reasons:

An intense focus on content helps teachers

• **understand the nature of the discipline.** Content immersion helps teachers balance the prevailing view of mathematics in school, i.e., a set of facts and procedures to be mastered, with the view that mathematics is about solving problems and developing abstractions. It helps teachers move their own definition of mathematical expertise away from "what you know" toward "what you can figure out." One participant put it this way: "Mathematics, especially in high school, is too frequently about *what* instead of *why*, and I think that programs like this, which force students and teachers to find deeper understandings of topics, do a great job of restoring the *why* to mathematics teaching . . . ."

• **experience mathematics as a student does.** We hear this over and over from participants in content-based programs. It's extremely difficult for an adult, comfortable in that part of the discipline covered in this or that curriculum, to recall the struggles and the confusion that come when one is first learning a mathematical topic. Many teachers are baffled at student misunderstanding of what seem like very simple ideas; they forget that the ideas are simple only after one has made them his or her own. As one teacher put it: "I can't think of a better way to enhance my teaching ability than to periodically put myself into the role of a student, to learn how we learn."

• **learn to connect ideas.** One of the most serious problems with precollege mathematics is the way it is segmented into discrete and disconnected topics. In content immersion experiences like BRC, MST, and PROMYS, teachers are forced to synthesize topics, to use algebra *and* geometry to solve a problem in probability, and to look for instances of a method (like the Euclidean algorithm) in situations which make no mention of it. Here's how one teacher put it: "After solving a problem or proving a theorem, I almost always ask myself, 'So where does this fit into the big picture?'…And I ask the same questions in my own classroom. Quite often, my students are not done when they have solved the problem on the board. This is because my next question is, 'So what does this mean?' or 'How does this relate to our previous problem?' or 'What do you think I'm going to ask next?'"

• **develop a knack for "mining" ideas.** Watching teachers work in one of these programs, you see that they quickly form collaborative groups—not in any formal sense, but as ways to help each other with the mathematics. And they soon realize that their colleagues often have good but half-formed ideas about how to approach problems. We've seen teachers in groups like this learn to tease out the germs of insights that are often implicit in hunches and conjectures. This skill for helping people refine and improve their own ideas is one of the most valuable things a teacher can bring to the classroom, and collaborative work in an immersion experience is a very effective way to develop it.

• **(re-)kindle a passion for mathematics.** Think about when you were in school. The teachers who made their subjects come alive were the teachers who were in love with their subjects. The best music teachers were the ones who played. The best English teachers were the ones who wrote. And the best mathematics teachers were the ones who were always thinking about this or that problem. There's simply no substitute for the excitement a teacher brings to class when he or/she is immersed in what he/she is teaching. "I can go on and on about what learning mathematics does to my teaching. But the most important thing for me is that the more math I learn, the more I learn to love math. And hopefully, my enthusiasm for math rubs off onto my students."

There's surely more involved in effective teaching than staying active in one's discipline. But we are convinced that an immersion experience goes a long way towards helping teachers help their students see mathematics as something they can do and something that can be immensely satisfying. The articles in this issue give three examples of how to make this happen.

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The Mathematicians and Education Reform Forum

The **Mathematicians and Education Reform (MER) Forum** seeks the effective participation of mathematicians in mathematics education reform at the K-12, undergraduate, and graduate levels, and the recognition of the importance of these efforts to the well being of the mathematics community. The MER Forum envisages the pursuit of educational reform through informed discussion of educational issues, thoughtful responses to changing educational conditions, and the promotion of exemplary programs. The creation and support of a network of mathematicians with a sustained commitment to mathematics education is central to this vision.

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