

Math 215
Homework 2
Solutions to selected exercises

Use properties **R1** – **R11** of the real numbers for the following exercises. You may also cite examples that we did in class. Be sure to justify every step.

1. Prove that

$$-(-x) = x$$

for all x .

By R6, $-(-x)$ is the unique value such that

$$-(-x) + (-x) = 0.$$

On the other hand, R6 also tells us that $x + (-x) = 0$. By uniqueness, we conclude that $x = -(-x)$.

2. Recall that the fraction x/y is by definition the product $x \cdot \frac{1}{y}$, which is defined whenever $y \neq 0$. Prove that

$$\frac{a}{b} = \frac{(ac)}{(bc)}$$

if $b, c \neq 0$.

By definition, $(ac)/(bc) = (ac) \cdot \frac{1}{bc}$. By an example in class, $1/(bc) = 1/b \cdot 1/c$. Furthermore,

$$(ac) \cdot \left(\frac{1}{b} \cdot \frac{1}{c}\right) = a \cdot \left(c \cdot \frac{1}{b}\right) \cdot \frac{1}{c} = a \cdot \left(\frac{1}{b} \cdot c\right) \cdot \frac{1}{c} = \left(a \cdot \frac{1}{b}\right) \cdot \left(c \cdot \frac{1}{c}\right)$$

by applying R2 and R1. By R7, $c \cdot (1/c) = 1$, and by R5, $(a \cdot 1/b) \cdot 1 = a \cdot 1/b$. Putting together this chain of equalities, we obtain

$$\frac{(ac)}{(bc)} = \frac{a}{b}$$

as desired.