

Math 215
Homework 3
Due Friday, September 12

Read sections 4 and 5. Exercises from the text: 4.1, 4.4, 4.5, 4.6, 4.7, 5.1, 5.2, 5.5

To turn in:

Use properties **R1** – **R11** of the real numbers for the following exercises. You may also cite examples that we did in class or previous homework exercises. Be sure to justify every step.

1. Recall that a real number is rational if it can be expressed as a ratio p/q , where p and q are integers and $q \neq 0$. A real number is irrational if it is not rational. State and prove the **contrapositive** of the following implication:

$$(a \text{ rational and } b \text{ irrational}) \text{ or } (a \text{ irrational and } b \text{ rational}) \implies a + b \text{ is irrational .}$$

2. Prove the following statement by **contradiction**:

$$\sqrt[3]{2} \text{ is irrational}$$

By definition, $\sqrt[3]{2}$ is the unique real number such that $(\sqrt[3]{2})^3 = 2$.

3. Prove:

(a) $a < b$ and $c < d \implies a + c < b + d$

(b) $x > 0 \iff \frac{1}{x} > 0$

4. Consider the following implication:

If x is rational and y is irrational, then xy is irrational.

(a) State the contrapositive.

(b) State the negation.

(c) Is the statement true or false? If true, provide a proof. If false, provide a counterexample.