Math 215
Homework 4
Due Friday, September 19

Exercises from the text: 5.4, 5.6, 5.7 and Problems I (p.53) #5, 14, 20

To turn in:

1. Prove that

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for all positive integers n.

2. For integers $0 \le k \le n$, we define the binomial coefficients by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

if $k \neq 0, n$, and

$$\binom{n}{n} = \binom{n}{0} = 1.$$

(a) Prove that

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

when $0 < k \le n$. Your proof does *not* require an induction argument.

(b) Use part (a) to prove (by induction on n) that $\binom{n}{k}$ is an integer, for any $0 \le k \le n$.

3. Read the proof of the Binomial Theorem in your textbook (Chapter 12, Theorem 12.3.1). Use the theorem to prove

$$\sum_{j=0}^{n} \binom{n}{j} = 2^n$$

and

$$\sum_{\substack{0 \le j \le n \\ j \text{ odd}}} \binom{n}{j} = 2^{n-1}$$

for all positive integers n.