Math 215
Homework 6
Due Friday, October 10

Read sections 8 and 9. Exercises from the text: 8.1, 8.2, 8.3, 9.1, 9.2, 9.3, 9.4

To turn in:

1. Let X be a set. For each subset $Y \subset X$, define the *characteristic function* of Y to be the function $\chi_Y : X \to \{0, 1\}$ defined by

$$\chi_Y(x) = \begin{cases} 0 & \text{if } x \notin Y \\ 1 & \text{if } x \in Y \end{cases}$$

Fix two subsets A and B of X.

(1) Show that the function $x \mapsto \chi_A(x) \cdot \chi_B(x)$ is the characteristic function of $A \cap B$.
(2) Find the subset C whose characteristic function is given by

$$\chi_C(x) = \chi_A(x) + \chi_B(x) - \chi_A(x) \cdot \chi_B(x).$$

2. Determine which of the following functions $f : \mathbb{R} \to \mathbb{R}$ are injective, which are surjective and which are bijective. Write down an inverse function for each of the bijective ones.

(1) $f(x) = x - 1$
(2) $f(x) = x^3$
(3) $f(x) = x^3 - x$
(4) $f(x) = e^x$

3. Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Show that if f and g are surjective, then $g \circ f$ is also surjective.