

Math 215
Homework 7
Due Friday, October 17

Problems from the text (II, pp. 115–119): 1, 11, 14, 17

To turn in:

A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is *bounded* if there exists a real number $M > 0$ so that $|f(x)| \leq M$ for all $x \in \mathbb{R}$.

1. Prove that the function

$$b(x) = \frac{1}{1+x^2}$$

is bounded.

2. Prove that the function

$$s(x) = x^2$$

is not bounded.

A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$. The function is *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$. For example, the identity function $f(x) = x$ is odd, while $f(x) = \cos(x)$ is even.

3. Prove or provide a counterexample to the following three statements:

- (a) If f and g are both even functions, then $f + g$ is even.
- (b) If f is odd and g is even, then $f + g$ is an odd function.
- (c) For every function f , there exists a function g so that $f + g$ is odd.

4. (**Extra Credit**) Prove that every function $f : \mathbb{R} \rightarrow \mathbb{R}$ can be expressed as a sum

$$f(x) = E(x) + O(x)$$

where $E : \mathbb{R} \rightarrow \mathbb{R}$ is an even function and $O : \mathbb{R} \rightarrow \mathbb{R}$ is an odd function.

Hint: Try showing that E and O are uniquely determined by f ; this might help you find a solution to 4.