

Math 215
Properties of the real numbers

R1. Commutativity

$$x + y = y + x$$
$$xy = yx$$

R2. Associativity

$$(x + y) + z = x + (y + z)$$
$$(xy)z = x(yz)$$

R3. Distributivity

$$x(y + z) = xy + xz$$

R4. Zero

$$x + 0 = 0 + x = x$$

R5. One

$$x \cdot 1 = 1 \cdot x = x$$

R6. Additive inverse

For each x , there is a unique real number $-x$ such that

$$x + (-x) = (-x) + x = 0$$

R7. Multiplicative inverse

For each $x \neq 0$, there is a unique real number $1/x$ such that

$$x \cdot \frac{1}{x} = \frac{1}{x} \cdot x = 1$$

R8. Ordering

For every pair of real numbers x and y , exactly one of the following holds:

- (1) $x < y$
- (2) $x = y$
- (3) $x > y$

R9. Addition law

For all x, y , and z ,

$$x < y \iff x + z < y + z$$

R10. Multiplication law

If $z > 0$, then

$$x < y \iff xz < yz$$

If $z < 0$, then

$$x < y \iff xz > yz$$

R11. Transitivity

$$x < y \text{ and } y < z \implies x < z$$