

Solutions to HW 1

1. If both m and n are odd, then the number of squares to be covered is mn which is odd. Each domino covers 2 squares, so the total number of squares covered by the dominoes must be even. Hence a perfect cover is impossible. On the other hand, if one of m or n , say m , is even, then we could place all dominoes vertically and this is a valid perfect covering.
2. We proceed by induction on $m + n$. If both m and n are 1 or 3, then it can be checked by hand, so assume that $m \geq 5$. Then the deleted white square is outside the top two rows, or outside the bottom two rows. Remove the two rows from above that it is outside. The removed rows form a 2-by- n board, so by problem 1 they can be tiled. The remaining board is $(m - 2)$ -by- n , $m - 2$ is odd, and its top left square is again white, so by induction we can cover it perfectly.
3. No, the prisoner cannot obtain his freedom. Assume he starts in a cell colored white. Then his goal is to reach the opposite cell, which is also white. Any route he chooses will alternate between black and white squares, since adjacent squares have differing colors. Since he starts and ends at white, this means that he would have traversed 33 white colored cells and 31 black colored cells. But this contradicts the fact that there are 32 cells of each color.
25. Say that a domino is *pierced* by a cut if it is cut in half. Clearly every domino is pierced by exactly one cut. The number of cuts is 10, and the number of dominoes is $18 < 20$, so there must be a cut C that pierces fewer than 2 dominos. If C pierces exactly 1 domino, then each of the two sub-boards formed by C has an even number of squares, and one of these squares is covered by the pierced domino. Hence the remaining odd number of squares (in each sub-board) are perfectly covered by dominoes (otherwise C would pierce more than one domino). But this is impossible. Therefore C pierces no domino, and it is a fault-line.
38. Among all pairings, choose one that minimizes that sum of the lengths of the line segments. Suppose that BR and $B'R'$ intersect at P . Then replace these segments by $B'R$ and BR' . By the triangle inequality $|B'P| + |PR| > |B'R|$ and $|BP| + |PR'| > |BR'|$. Adding these two equations results in $|BR| + |B'R'| > |BR'| + |B'R|$ which contradicts our choice.
43. No, the middle cube needs six cuts to separate it, one for each of its sides.