# Randomizations

$T$ – classical first order theory
$T^R$ – the randomization of $T$

The models of $T^R$ are essentially spaces of $\mathcal{M}$-valued random variables where $\mathcal{M} \models T$.

We study $T^R$ in the framework of continuous logic.
Idea: Instead of being true or false, sentences are assigned a probability in $[0,1]$.

# NIP

Classical Version:
$\phi(x,y)$ is NIP $\Longleftrightarrow$ for any indiscernible sequence $(a_i : i \in \mathcal{I})$ and tuple $b$, there is some end segment $\mathcal{I}_0 \subseteq \mathcal{I}$ and $\eta \in \{0,1\}$ such that $\phi(a_i, b)^\eta$ holds for all $i \in \mathcal{I}_0$.

Continuous Version:
$\phi(x,y)$ is NIP $\Longleftrightarrow$ for any indiscernible sequence $(a_i : i \in \mathcal{I})$ and tuple $b$, there is $L \in [0,1]$ such that $\forall \varepsilon > 0$, there is some end segment $\mathcal{I}_0 \subseteq \mathcal{I}$ such that $|\phi(a_i,b) - L| < \varepsilon$ for all $i \in \mathcal{I}_0$.

## Known

T is ________ if and only if $T^R$ is ________.

- $\omega$-categorical
- $\omega$-stable
- stable
- NIP

But if T is simple and unstable, $T^R$ is not simple.

## Questions

- Do randomizations preserve various notions of strong dependence, such as dp-minimality, VC-density one, and VC-minimality?
- Are these notions interesting in the continuous setting?
- More generally, what happens to dp-rank when we randomize?
- Do the continuous versions of these notions interact in an analogous way to the classical versions?