Quiz 13

MATH 210, CALCULUS III, SUMMER 2015

NAME:

Problem 1. Let C be the parabola $\mathbf{r}(t) = \langle 4t, t^2 \rangle$ for $0 \le t \le 1$. Find the circulation of $\mathbf{F} = \langle x, y \rangle$ on C.

$r'(t) = \langle 4, 2t \rangle \qquad F(r(t)) = \langle 4t, t^2 \rangle$

Circulation: $\int_C F \cdot dr = \int_0^1 \langle 4t, t^2 \rangle \cdot \langle 4, 2t \rangle \, dt$

$= \int_0^1 16t + 2t^3 \, dt = \frac{16t^2}{2} + \frac{2t^4}{4} \Big|_0^1 = 8t^2 + \frac{1}{2}t^4 \Big|_0^1$

$= 8(1)^2 + \frac{1}{2}(1)^4 - (8(0)^2 + \frac{1}{2}(0)^4) = 8 + \frac{1}{2} = \frac{17}{2}$

Problem 2. Show that $\mathbf{F} = \langle yz, xz, xy \rangle$ is conservative on $\mathbb{R}^3$.
Extra credit (1 point): Find a potential function ϕ such that $\nabla \phi = \mathbf{F}$.

$\frac{\partial}{\partial y}(yz) = z = \frac{\partial}{\partial x}(xz) \qquad \frac{\partial}{\partial z}(yz) = y = \frac{\partial}{\partial x}(xy) \qquad \frac{\partial}{\partial z}(xz) = x = \frac{\partial}{\partial y}(xy)$

Extra Credit:

$\int yz \, dx = xyz + C(y, z) = \phi$

$\frac{\partial}{\partial y}(xyz + C(y,z)) = xz + C_y(y,z) = xz$

$C_y(y, z) = 0$

$C_y(y,z) = \int 0 \, dy = 0 + d(z) = d(z)$

So $\phi = xyz + d(z)$

$\frac{\partial}{\partial z}(xyz + d(z)) = xy + d'(z) = xy$

$d'(z) = 0$

$d(z) = C \quad$ Let $C = 0$

$\boxed{\phi = xyz}$