Quiz 5

MATH 210, CALCULUS III, SUMMER 2015

NAME:

Problem 1. Find the length of the polar curve $r = \theta^2$ where $0 \leq \theta \leq 2\pi$.

$f(\theta) = \theta^2 \quad f'(\theta) = 2\theta$

$$L = \int_0^{2\pi} \sqrt{(\theta^2)^2 + (2\theta)^2}\, d\theta = \int_0^{2\pi} \sqrt{\theta^4 + 4\theta^2}\, d\theta$$

$$= \int_0^{2\pi} \theta\sqrt{\theta^2+4}\, d\theta \qquad \text{Let } u = \theta^2+4 \qquad 0^2+4 = 4$$

$$du = 2\theta\, d\theta \qquad (2\pi)^2+4 = 4\pi^2+4$$

$$\tfrac{1}{2}\, du = \theta\, d\theta$$

$$= \frac{1}{2}\int_4^{4\pi^2+4} \sqrt{u}\, du$$

$$= \frac{1}{2}\cdot\frac{2}{3}\, u^{3/2}\Big|_4^{4\pi^2+4} = \boxed{\frac{1}{3}(4\pi^2+4)^{3/2} - \frac{1}{3}(4)^{3/2}} = \frac{8}{3}\left((\pi^2+1)^{3/2} - 1\right)$$

Problem 2. Find the equation of the plane Q containing the point $(1, 1, 0)$ parallel to the plane $R : 3x - 2y + z = 5$.

$$3(x-1) - 2(y-1) + z = 0$$

$$3x - 2y + z = 1$$