

STATEMENT OF RESEARCH

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My research interests lie in the field of birational algebraic geometry and commutative algebra. In my work I use various algebraic and geometric tools including those from the theory of resolution of singularities, valuation theory, and analysis on toric varieties. Over the past several years I worked on global and local toroidalization problems. My results fall under the following two categories:

- 1) Toroidalization of strongly prepared morphisms of n -folds to surfaces;
- 2) Toroidalization of generating sequences in dimension 2 function fields.

Although purely abstract most of the arguments used in my work are constructive and therefore can be programmed on a computer. Let me now describe my results in more detail.

- 1) Given a dominant morphism $f : X \rightarrow Y$ of varieties over a field k , one can state the toroidalization problem as follows.

Does there exist a commutative diagram

$$\begin{array}{ccc} X_1 & \xrightarrow{f_1} & Y_1 \\ \downarrow & & \downarrow \\ X & \xrightarrow{f} & Y \end{array}$$

such that the vertical maps are products of blowups of nonsingular subvarieties and $f_1 : X_1 \rightarrow Y_1$ is toroidal? A dominant morphism $f_1 : X_1 \rightarrow Y_1$ of nonsingular varieties with fixed simple normal crossing divisors D_X on X_1 and E_Y on Y_1 is said to be toroidal if locally it is a map of toric varieties with the tori defined by $X_1 - D_X$ and $Y_1 - E_Y$.

Toroidal morphisms are monomial, that is locally given by a collection of monomials, but this description is not complete. Another way to define toroidal morphisms would be by means of logarithmic differentials.

Theorem 1. *Suppose that k is an algebraically closed field of characteristic 0 and f is a strongly prepared morphism from n -fold to surface. Then f can be toroidalized.*

The notion of strongly prepared morphisms from n -folds to surfaces is defined in [2]. Being strongly prepared is a local property of morphism f , so the definition in [2] contains a list of all admissible local equations of f . One can also obtain a very nice description of strongly prepared morphisms using logarithmic differentials.

2) A local version of the toroidalization problem can be stated as follows.

Suppose that $R \subset S$ are regular local rings, essentially of finite type over a field k , such that the quotient field K^* of S is a finite extension of the quotient field K of R . Let V^* be a valuation ring of K^* which dominates S and let V be the restriction of V^* to K . Do there exist sequences of monoidal transforms $R \rightarrow R'$ and $S \rightarrow S'$ such that V^* dominates S' , S' dominates R' and there are regular parameters (x_1, x_2, \dots, x_n) in R and (y_1, y_2, \dots, y_n) in S' , units $\delta_1, \delta_2, \dots, \delta_n$ in S' and a nonsingular matrix (a_{ij}) of nonnegative integers such that

$$\begin{aligned} x_1 &= y_1^{a_{11}} y_2^{a_{12}} \cdots y_n^{a_{1n}} \delta_1 \\ &\vdots \\ x_n &= y_1^{a_{n1}} y_2^{a_{n2}} \cdots y_n^{a_{nn}} \delta_n. \end{aligned}$$

The local monomialization can be achieved in all dimensions over algebraically closed characteristic 0 field k ([1],[3]). It also holds in positive characteristic for dimension 2 function fields when V^*/V is defectless ([3]). The next theorem extends the results of Cutkosky and Piltant and shows that in characteristic 0 the map between generating sequences of valuations of dimension 2 function fields has a toroidal structure.

Theorem 2. *Let k be an algebraically closed field of characteristic 0, and let K^*/K be a finite extension of algebraic function fields of transcendence degree 2 over k . Let ν^* be a k -valuation of K^* with valuation ring V^* , and let ν be the restriction of ν^* to K . Suppose that $R \rightarrow S$ is an extension of algebraic regular local rings with quotient fields K and K^* respectively, such that V^* dominates S and S dominates R . Then there exist sequences of quadratic transforms $R \rightarrow \bar{R}$ and $S \rightarrow \bar{S}$ along ν^* such that \bar{S} dominates \bar{R} and one of the following holds*

- 1) ν^* and ν are divisorial valuations. Then \bar{R} and \bar{S} are discrete valuation rings with regular parameters $x \in \bar{S}$ and $u \in \bar{R}$ such that $u = x^a \delta$ for some unit $\delta \in \bar{S}$. We also have that $\{x\}$ is a minimal generating sequence of ν^* and $\{u\}$ is a minimal generating sequence of ν .

- 2) ν^* and ν are valuations of rational rank 2. Then there exist regular parameters (x, y) in \bar{S} and (u, v) in \bar{R} such that $\{x, y\}$ is a minimal generating sequence of ν^* , $\{u, v\}$ is a minimal generating sequence of ν and

$$\begin{aligned} u &= x^a y^b \delta_1 \\ v &= x^c y^d \delta_2 \end{aligned}$$

for some units $\delta_1, \delta_2 \in \bar{S}$ and some nonnegative integers a, b, c, d with $ad - bc \neq 0$.

- 3) ν^* and ν are nondiscrete valuations of rational rank 1. Then there exists a minimal generating sequence $\{H_i\}_{i \geq 0}$ of ν^* in \bar{S} such that $\{H_0^b \delta, \{H_i\}_{i > 0}\}$ is a generating sequence in \bar{R} , $\delta \in \bar{S}$ is a unit and b is a nonnegative integer.
- 4) ν^* and ν are discrete but not divisorial valuations. Then there exist regular parameters (x, y) in \bar{S} and (u, v) in \bar{R} such that $\nu^*(K^*)$ is generated by $\nu^*(x)$, $\nu(K)$ is generated by $\nu(u)$ and $u = x^a \delta$ for some unit $\delta \in \bar{S}$. Moreover, \bar{S} has a non-minimal generating sequence $\{x, \{T_i\}_{i > 0}\}$ such that $\{u, \{T_i\}_{i > 0}\}$ forms a non-minimal generating sequence in \bar{R} .

CURRENT AND FUTURE RESEARCH

The theory of valuations centered at 2 dimensional regular local rings is very much developed. There are several approaches to understanding a valuation of a function field of a surface: geometric, by specifying the sequence of blowups of points; combinatorial, through the dual graph of a valuation; and algebraic, by the Puiseux exponents of a valuation. Generating sequence of a valuation is another powerful tool in dimension 2. However, not much is known about higher dimensional functional fields.

Recently I have become very interested in investigating valuations centered in regular local rings of dimension 3. One can ask a number of questions about valuation ν in dimension higher than 2. If by analogy with the surface case, the dual graph of a valuation in a 3-fold has been defined, will it capture all the properties of ν ? Is it possible to find an algorithm for constructing a generating sequence of ν ? What is the relationship between valuations of regular local rings and valuations of their completions?

I also have an ongoing project with Laura Ghezzi on extending our proof of toroidalization of generating sequences in dimension 2 function fields to positive characteristic.

REFERENCES

- [1] S.D. Cutkosky, *Local factorization and monomialization of morphisms*, *Asterisque* 260, 1999.
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- [3] S. D. Cutkosky, O. Piltant, *Ramification of Valuations*, *Adv. in Math.* 183 (2004), 1-79.