MATH 121 **Hour Exam I Solution** **Radford 09/27/02**

1. (10 points): $y = y_1 = x^4 + 2$. Translate graph of y_1 to the left 7 units which results in $y_2 = (x+7)^4 + 2$ (**5 points**). Now translate down 15 units which results in $y_3 = y_2 - 17 = ((x+7)^4+2) - 15$. Answer: $g(x) = ((x+7)^4+2) - 15$ (**5 points**) or $g(x) = (x+7)^4 - 13$.

2. (10 points): $y = f(x) = \dfrac{3x-8}{2x+5}$. Therefore $y(2x+5) = 3x - 8$ (**3 points**) from which $x(2y-3) = -8 - 5y$ follows and consequently $x = \dfrac{5y+8}{-2y+3}$ (**4 points**). From the last equation we deduce $f^{-1}(x) = \dfrac{5x+8}{-2x+3}$ (**3 points**).

3. (20 points): Since 2 is a root of $p(x) = x^3 - 5x^2 + 2x + 8$ it follows that $x - 2$ is a factor of $p(x)$. Dividing $x - 2$ into $p(x)$ gives a quotient of $x^2 - 3x - 4$ and remainder 0 (**7 points**). Therefore $p(x) = x^3 - 5x^2 + 2x + 8 = (x-2)(x^2-3x-4) = (x-2)(x-4)(x+1)$. Conclusion: a) the roots are $2, 4, -1$ (**7 points**) and b) a factorization of $p(x)$ into linear factors is $(x-2)(x-4)(x+1)$ (**6 points**) .

4. (15 points): Let x be a base length, let h be the height, let S be the surface area, and let V be the volume. Then $108 = S = x^2 + 4xh$ since there is no top, and

$$V = x^2 h = x^2 \left(\frac{108 - x^2}{4x}\right) = \frac{1}{4}(108x - x^3), \quad x \geq 0, \quad (\textbf{5 points})$$

Here you need your calculator to find where the volume $V = V(x)$ is maximized. Answer: $x = 6$ (**5 points**). From the equation $108 = x^2 + 4xh = 36 + 24h$ we deduce that $h = 3$ (**5 points**).

5. (15 points): Let $p(x) = 3x^6 + 6x^5 + 2x^4 + 4x^3 + 4x + 8$. Then remainder when $p(x)$ is divided by $x + 2 = x - (-2)$ is $p(-2) = 3(-2)^6 + 6(-2)^5 + 2(-2)^4 + 4(-2)^3 + 4(-2) + 8 = 3(2^6) - 3(2^6) + 2^5 - 2^5 - 8 + 8 = 0$. Therefore a) the remainder is 0 (**10 points**) which means that b) $x - (-2) = x + 2$ is a factor of $p(x)$ (**5 points**).

6. (10 points): Let $p(x) = a(x-(-3))(x-(-1))(x-4) = a(x+3)(x+1)(x-4)$, where a is a real number (**3 points**). Then $11 = p(5) = a(5+3)(5+1)(5-4) = a48$ (**3 points**) means that $a = 11/48$. Thus $p(x) = \dfrac{11}{48}(x+3)(x+1)(x-4)$ is an answer (**4 points**).

7. (20 points): $v_0 = -64$ and $h_0 = 1200$. Therefore $h(t) = -16t^2 + v_0 t + h_0 = -16t^2 - 64t + 1200$ (**7 points**). The object hits the ground when $h(t) = 0$, $t \geq 0$. Thus $-16t^2 - 64t + 1200 = 0$, or $t^2 + 4t - 75 = 0$. By the quadratic formula $t = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-75)}}{2} = \dfrac{-4 \pm \sqrt{316}}{2}$, so a) $t = -2 + \sqrt{79} \approx 6.89$ (**7 points**). For b) we solve $h(t) = 500$, or $-16t^2 - 48t + 700 = 0$, or $4t^2 + 12t - 175 = 0$ which gives $t = \dfrac{-12 + \sqrt{2944}}{8} \approx 5.28$. (**6 points**)