MATH 121 **Hour Exam I Solution** **Radford 09/27/02**

1. (10 points): $y = y_1 = x^3 + 5$. Translate graph of y_1 to the left 13 units which results in $y_2 = (x+13)^3+5$ (**5 points**). Now translate down 17 units which results in $y_3 = y_2 - 17 = ((x+13)^3+5)-17$. Answer: $g(x) = ((x+13)^3+5)-17$ (**5 points**) or $g(x) = (x+13)^3-12$.

2. (10 points): $y = f(x) = \dfrac{9x-4}{7x+6}$. Therefore $y(7x+6) = 9x-4$ (**3 points**) from which $x(7y-9) = -4-6y$ follows and consequently $x = \dfrac{6y+4}{-7y+9}$ (**4 points**). From the last equation we deduce $f^{-1}(x) = \dfrac{6x+4}{-7x+9}$ (**3 points**).

3. (20 points): Since 3 is a root of $p(x) = x^3 - 2x^2 - 33x + 90$ it follows that $x-3$ is a factor of $p(x)$. Dividing $x-3$ into $p(x)$ gives a quotient of x^2+x-30 and remainder 0 (**7 points**). Therefore $p(x) = x^3-2x^2-33x+90 = (x-3)(x^2+x-30) = (x-3)(x-5)(x+6)$. Conclusion: a) the roots are $3, 5, -6$ (**7 points**) and b) a factorization of $p(x)$ into linear factors is $(x-3)(x-5)(x+6)$ (**6 points**) .

4. (15 points): Let x be a base length, let h be the height, let S be the surface area, and let V be the volume. Then $75 = S = x^2 + 4xh$ since there is no top, and

$$V = x^2h = x^2\left(\frac{75-x^2}{4x}\right) = \frac{1}{4}(75x - x^3), \quad x \geq 0, \quad (\textbf{5 points})$$

Here you need your calculator to find where the volume $V = V(x)$ is maximized. Answer: $x = 5$ (**5 points**). From the equation $75 = x^2 + 4xh = 25 + 20h$ we deduce that $h = 2.5$ (**5 points**).

5. (15 points): Let $p(x) = 3x^6+9x^5+4x^2+4x-24$. Then remainder when $p(x)$ is divided by $x+3 = x-(-3)$ is $p(-3) = 3(-3)^6+9(-3)^5+4(-3)^2+4(-3)-24 = 3^7-3^7+36-12-24 = 0$. Therefore a) the remainder is 0 (**10 points**) which means that b) $x-(-3) = x+3$ is a factor of $p(x)$ (**5 points**).

6. (10 points): Let $p(x) = a(x-(-4))(x-(-2))(x-5) = a(x+4)(x+2)(x-5)$, where a is a real number (**3 points**). Then $13 = p(6) = a(6+4)(6+2)(6-5) = a80$ (**3 points**) means that $a = 13/80$. Thus $p(x) = \dfrac{13}{80}(x+4)(x+2)(x-5)$ is an answer (**4 points**).

7. (20 points): $v_0 = -48$ and $h_0 = 800$. Therefore $h(t) = -16t^2+v_0t+h_0 = -16t^2-48t+800$ (**7 points**). The object hits the ground when $h(t) = 0$, $t \geq 0$. Thus $-16t^2-48t+800 = 0$, or $t^2+3t-50 = 0$. By the quadratic formula $t = \dfrac{-3 \pm \sqrt{3^2-4(1)(-50)}}{2} = \dfrac{-3 \pm \sqrt{209}}{2}$, so a) $t = \dfrac{-3+\sqrt{209}}{2} \approx 5.73$ (**7 points**). For b) we solve $h(t) = 200$, or $-16t^2-48t+600 = 0$, or $2t^2+6t-75 = 0$ which gives $t = \dfrac{-6+\sqrt{636}}{4} \approx 4.80$. (**6 points**)