MATH 215 # Hour Exam II **Radford** **11/18/05**

Name (print) ______________________________

(1) There are *four problems* on this exam. Problem 4 is on the back. (2) *Return* this exam copy with your test booklet. (3) *You are expected to abide by the University's rules concerning academic honesty.*

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1. (30 pts.) Let A and B be sets.

   a) Define the sets $A \cap B$ and $A \cup B$ using conditional definitions.

   b) What does it mean for $x \notin A \cap B$? For $x \notin A \cup B$?

   c) Let U be a universal set and $A, B \subseteq U$. Show that $A = (A \cap B) \cup (A \cap B^c)$ and that the union is disjoint.

   d) Suppose that $A = \{1, 0, \pi\}$. Compute $P(A)$.

2. (20 pts.) For sets $A_1, \ldots, A_n$ we define the union $A_1 \cup \cdots \cup A_n$ and intersection $A_1 \cap \cdots \cap A_n$ inductively by

$$A_1 \cup \cdots \cup A_n = \begin{cases} A_1 & : \quad n = 1; \\ (A_1 \cup \cdots \cup A_{n-1}) \cup A_n & : \quad n > 1 \end{cases}$$

   and

$$A_1 \cap \cdots \cap A_n = \begin{cases} A_1 & : \quad n = 1; \\ (A_1 \cap \cdots \cap A_{n-1}) \cap A_n & : \quad n > 1 \end{cases}.$$

   Now suppose that U is a universal set and $A_1, \ldots, A_n \subseteq U$. Use De Morgan's Laws and the definitions above to construct a proof by induction that

$$(A_1 \cup \cdots \cup A_n)^c = A_1^c \cap \cdots \cap A_n^c.$$

   [Comment: You may assume $A_1 \cup \cdots \cup A_m, A_1 \cap \cdots \cap A_m \subseteq U$ for all $A_1, \ldots, A_m \subseteq U$. *The steps of your proof must at least be implicitly justified.*]

3. (25 pts.) Let $f : A \longrightarrow B$ be a function and suppose $X, Y \subseteq A$.

   a) Show that $\overrightarrow{f}(X \cap Y) \subseteq \overrightarrow{f}(X) \cap \overrightarrow{f}(Y)$. [Comment: $\overrightarrow{f}(X) = \{f(x) \,|\, x \in X\} = f(X)$, the latter notation was used in class.]

   b) Suppose that f is an injection. Show that $\overrightarrow{f}(X \cap Y) = \overrightarrow{f}(X) \cap \overrightarrow{f}(Y)$.

   c) Now suppose that $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7\}$, $X = \{1, 2\}$, and $Y = \{2, 3\}$. Find a surjection $g : A \longrightarrow B$ such that $\overrightarrow{g}(X \cap Y) \subset \overrightarrow{g}(X) \cap \overrightarrow{g}(Y)$. *Justify your answer.*

4. (25 pts.) A committee of 7 is to be formed from a group of 10 people. At least one of two individuals A and B is to be on the committee. Let X be the set of those committees of 7 which include A and let Y be the set of those committees of 7 which include B.

   a) Determine $|X|$ and $|Y|$ explicitly.

   b) Determine explicitly the number of the committees of 7 which include both A and B. Express the set of these committees in terms of X and Y.

   c) Use the inclusion-exclusion principle to determine explicitly the number of committees of 7 which include either A or B (perhaps both). Express the set of these committees in terms of X and Y.