MATH 215 **Summer 2009** **Radford**

Written Homework #2 (Revised)

Due at the beginning of class 06/26/2009

You may assume the propositions and theorems of the text. You may assume that the product of two positive real numbers, and the product of two negative real numbers, is positive. You may also assume that the product of a positive real number and a negative real number is negative.

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1. Let n be an integer. Consider the assertion: If $\boxed{1 < n < 3}$ then $n^3 < 2n^2 + 15n$.

   a) Prove the assertion by cases.

   b) Prove the assertion by "working backwards".

   c) Determine all integers n such that $n^3 < 2n^2 + 15n$. You must justify your answer.

2. Let a be a real number. Consider the assertion: $a^2 \geq 7a$ implies $a \leq 0$ or $a \geq 7$.

   a) Prove the assertion by contradiction.

   b) Prove the assertion directly.

3. Let a be a real number. Consider the assertion: $a^2 - 9a + 18 < 0$ implies $3 \leq a < 6$.

   a) Prove the assertion $\boxed{\text{by}}$ contradiction.

   b) Prove the assertion directly.

   c) Is the converse of the assertion true? Give a proof or counterexample.

An integer n is *even* if $n = 2m$ for some integer m and is *odd* if $n = 2m + 1$ for some integer m.

4. Let ℓ be an odd integer. Prove, by cases, that if $n^2 + \ell n$ is even for all integers n.

5. Let ℓ be an odd integer. Prove, by induction, that $n^2 + \ell n$ is even for all integers $n \geq 1$.