

Written Homework #3

Due at the beginning of class 07/06/2009

1. Let a_1, a_2, a_3, \dots be the terms of the Fibonacci sequence.
 - a) Show that $n = 6$ is the smallest positive integer such that $a_n \leq 2^{n-3}$.
 - b) Prove, by induction, that $a_n \leq 2^{n-3}$ for all $n \geq 6$.
2. Prove, by induction, that the sum of the squares of the first $m \geq 1$ odd integers is given by

$$1^2 + 3^2 + \dots + (2m - 1)^2 = \frac{m(2m - 1)(2m + 1)}{3}.$$

3. Let A and B be sets. Working from definitions, prove that $A = (A - B) \cup (A \cap B)$ and that $(A - B) \cap (A \cap B) = \emptyset$. (Thus A is the disjoint union of $A - B$ and $A \cap B$.)