MATH 215 **Summer 2009** **Radford**

Written Homework #4

Due at the beginning of class 07/10/2009

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1. For $n \geq 1$ we define the union of sets $A_1, \ldots, A_n$ inductively by

$$A_1 \cup \cdots \cup A_n = \begin{cases} A_1 & : \quad n = 1; \\ (A_1 \cup \cdots \cup A_{n-1}) \cup A_n & : \quad n > 1 \end{cases}.$$

Let $A, A_1, \ldots, A_n$ be sets, where $n \geq 1$. Prove, by induction, that

$$A \cap (A_1 \cup \cdots \cup A_n) = (A \cap A_1) \cup \cdots \cup (A \cap A_n).$$

[You may assume $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for sets A, B, C.]

2. Describe the power set $P(S)$ of S the following cases by listing its elements:
a) $S = \emptyset$; b) $S = \{41\}$; c) $S = \{\emptyset\}$; d) $S = \{\pi, e\}$.

3. Let A, B be sets. Complete the following truth table and use it to explain why A is the disjoint union of $A - B$ and $A \cap B$.

$x \in A$	$x \in B$	$x \in A - B$	$x \in A \cap B$	$x \in (A - B) \cap (A \cap B)$	$x \in A$

4. Let U be a universal set and $A, B \subseteq U$. Complete the following truth table

$x \in A$	$x \in B$	$x \in A \cup B$	$x \in (A \cup B)^c$	$x \in A^c$	$x \in B^c$	$x \in A^c \cap B^c$

and use it to explain why $(A \cup B)^c = A^c \cap B^c$, one of De Morgan's Laws.

5. Let U be a universal set and $A, B \subseteq U$. Using the fact that $A^{cc} = A$, deduce De Morgan's other law $(A \cap B)^c = A^c \cup B^c$ from his law $(A \cup B)^c = A^c \cap B^c$.