MATH 180 (16397) **Quiz 1 (Version I) 01/20/06;** **Solution 01/23/06**

Name (print) ______________________________ **Discussion hour (T Th ___)**

---

1. (8 pts.) Let $f(x) = \sqrt{x^2+2}$ and $g(x) = x^5 + 3$. Find:

   a) $(f \circ g)(x)$; **Solution**: $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)^2+2} = \sqrt{(x^5+3)^2+2}$. Thus $\boxed{(f \circ g)(x) = \sqrt{(x^5+3)^2+2}.}$ *4 points.*

   b) $(g \circ g)(x)$. **Solution**: $(g \circ g)(x) = g(g(x)) = g(x)^5 + 3 = (x^5+3)^5 + 3$. Thus $\boxed{(g \circ g)(x) = (x^5+3)^5+3.}$ *4 points.*

(Do *not* simplify.)

2. (12 pts.) A certain population grew exponentially at the annual rate of 1.92%. Initially, in 1928, the population was 194.

   a) Let $P(t)$ be the population at time t years ($t = 0$ in 1928). Determine the function $P(t)$. **Solution**: $P(t) = P(0)(1 + \frac{1.92}{100})^t = P(0)(1 + .0192)^t = 194(1.0192)^t$. Thus $\boxed{P(t) = 194(1.0192)^t.}$ *4 points.*

   b) Find the time it took the population to double. (Express your answer in terms of the natural logarithm). **Solution**: $2P(0) = P(t) = P(0)(1.0192)^t$ so $2 = (1.0192)^t$. Taking natural logarithms of both sides we have $\ln 2 = t(\ln 1.0192)$ so $\boxed{t = \frac{\ln 2}{\ln 1.0192}.}$ *4 points.*

   c) Find the *percentage continuous* growth rate. (Express your answer in terms of the natural logarithm). **Solution**: $P(0)(1.0192)^t = P(t) = P(0)e^{kt}$ for some real number k. With $t = 1$ we have $1.0267 = e^k$. Therefore $\ln 1.0192 = \ln e^k = k \ln e = k$. Thus $\boxed{k = \ln 1.0192.}$ *4 points.*

MATH 180 (16397) Quiz 1 (Version II) 01/20/06; Solution 01/23/06

Name (print) ______________________ **Discussion hour (T Th __)**

---

1. (8 pts.) Let $f(x) = \sqrt{x^3+5}$ and $g(x) = x^7 + 4$. Find:

a) $(f \circ g)(x)$; **Solution**: $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)^3+5} = \sqrt{(x^7+4)^3+5}$. Thus $\boxed{(f \circ g)(x) = \sqrt{(x^7+4)^3+5}.}$ *4 points*.

b) $(g \circ g)(x)$. **Solution**: $(g \circ g)(x) = g(g(x)) = g(x)^7 + 4 = (x^7+4)^7 + 4$. Thus $\boxed{(g \circ g)(x) = (x^7+4)^7+4.}$ *4 points*.

(Do *not* simplify.)

2. (12 pts.) A certain population grew exponentially at the annual rate of 2.76%. Initially, in 1892, the population was 251.

a) Let $P(t)$ be the population at time t years ($t = 0$ in 1892). Determine the function $P(t)$. **Solution**: $P(t) = P(0)(1 + \frac{2.76}{100})^t = P(0)(1 + .0276)^t = 251(1.0276)^t$. Thus $\boxed{P(t) = 251(1.0276)^t.}$ *4 points*.

b) Find the time it took the population to double. (Express your answer in terms of the natural logarithm). **Solution**: $2P(0) = P(t) = P(0)(1.0276)^t$ so $2 = (1.0276)^t$. Taking natural logarithms of both sides we have $\ln 2 = t(\ln 1.0276)$ so $\boxed{t = \frac{\ln 2}{\ln 1.0276}.}$ *4 points*.

c) Find the *percentage continuous* growth rate. (Express your answer in terms of the natural logarithm). **Solution**: $P(0)(1.0276)^t = P(t) = P(0)e^{kt}$ for some real number k. With $t = 1$ we have $1.0276 = e^k$. Therefore $\ln 1.0276 = \ln e^k = k \ln e = k$. Thus $\boxed{k = \ln 1.0276.}$ *4 points*.