

Short Writing Exercise #2 Revisited

The infinite series $1/2 + 1/4 + \cdots + 1/2^n = 1$.

Infinite series are very fascinating. They can be understood in terms of geometric figures such as squares that we are going to demostate in this exercise by considering a very natural example which involves powers of two: $1/2 + 1/4 + \cdots + 1/2^n = 1$, where n is inifintely large.

Take a square with four sides of equal length, say one. Now devide it up into two parts. Each part, therefore, has area $1/2$. Now label one part $1/2$. Take the other area and split into two parts. Clearly the other areas are $1/4$ and $1/4$. Label one of the parts $1/4$, since it has area $1/4$. Repeat the process on the unlabeled square. In this manner, if one would be able to go on forever, $1/2 + 1/4 + \cdots + 1/2^n + \cdots = 1$.

Another way of saying it again. Consider a square with area one. Split the square into two equal equal rectangles by a vertical line. Label the right rectangle by its area $1/2$; that is one-half the area of the original square. Divide the left rectangle into two equal rectangles by a horizontal line. They are squares. Label the lower lefthand square by its area $1/4$; that is one-fourth the area of the original square. Note the area of the labeled portions of the square is $1/2 + 1/4$. Now repeat the process to the square in the upper left hand corner. It's area is $(1/4)(1/2) + (1/4)(1/4) = 1/8 + 1/16$. Thus, continuing, clearly $1/2 + 1/4 + 1/8 + 1/16 + \cdots = 1$.