

# Short Writing Exercise #3

Due at class time 10/28/02

The purpose of this short exercise is to give a general description of exponential decay and then explain the details in the specific case of carbon dating. Base your writing on the general description of exponential decay below and on the specific text book problem which follows. There are some small problems – stylistic, logical, or linguistic – in the writing which follows. In particular sentences which begin with an italicized word need close scrutiny.

**Exponential decay:** Let  $C(t)$  be the amount of a given substance at time  $t \geq 0$ . Then the substance decays exponentially if the rate at which it diminishes at time  $t$  is proportional to the amount of substance present at that time; that is there exists a positive real number  $k$  such that  $C'(t) = -kC(t)$  for all  $t \geq 0$ . By calculus this equation is equivalent to

$$C(t) = C(0)e^{-kt} \quad (1)$$

for all  $t \geq 0$ .

Now  $e^x$  and  $2^x$  describe 1-1 functions whose ranges are  $(-\infty, \infty)$ . Thus  $e^{-k} = 2^{-1/K}$  for some unique positive real number  $K$ . Since

$$e^{-kt} = (e^{-k})^t = (2^{-1/K})^t = (2^{-1})^{t/K} = \left(\frac{1}{2}\right)^{t/K}$$

we may write

$$C(t) = C(0)(0.5)^{t/K}.$$

Since  $C(K) = C(0.5)^{K/K} = C(0)(0.5)$  it follows that  $K$  is the length of time it takes for the substance to diminish to one-half of what it was originally. Generally, at time  $t \geq 0$  the calculation  $C(t+K) = C(0)(.05)^{(t+K)/K} = C(0)(0.5)^{1+t/K} = (0.5)C(0)(0.5)^{t/K} = (0.5)C(t)$  shows that the amount at time  $t$  diminishes to one-half in time  $K$ . The time  $K$  is called the half-life of the substance.

**A specific textbook problem:** The radioactive carbon-14 isotope is present in small quantities in all life forms, and is constantly replenished until the organism dies, after which it decays to a stable carbon-12 at the rate proportional to the amount of carbon-14 present at the time  $t$ , with half life of 5730 years. In 1988 three teams of scientists found that the Shroud of Turin, which is reputed to be the burial cloth of Jesus, contained 91% of the amount of carbon-14 that a freshly made cloth of the same material. How old is the Shroud of Turin, according to these data?<sup>1</sup>

A finished, type-written draft is due at the beginning of class, Monday, 10/28/02.

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<sup>1</sup>From course materials used by B. Srinivasan, Fall 2001.