

**Problem 1.** Let  $y = f(x) = 6 + \sqrt{5x - 7}$ . The calculation

$$\begin{aligned} y &= 6 + \sqrt{5x - 7}, \\ y - 6 &= \sqrt{5x - 7}, \\ (y - 6)^2 &= 5x - 7, \\ (y - 6)^2 + 7 &= 5x, \\ \frac{(y - 6)^2 + 7}{5} &= x \quad \text{(15 points)} \end{aligned}$$

shows that the inverse of  $f(x)$  is given by  $f^{-1}(x) = \frac{(x - 6)^2 + 7}{5}$  (5 points).

**Problem 2.** Since  $f(x) = x^3 + 2x - 3$  has integer coefficients, the rational roots of  $f(x)$  are among  $\pm 1, \pm 3$ . Since  $f(1) = 0$  it follows that  $x = 1$  is a root of  $f(x)$ . Therefore  $x - 1$  divides  $f(x)$ . Dividing  $x - 1$  into  $f(x)$  gives the quotient of  $x^2 + x + 3$  (4 points) and remainder 0. Thus

$$x^3 + 2x - 3 = (x - 1)(x^2 + x + 3)$$

By the quadratic formula the roots of  $x^2 + x + 3$  are  $\frac{-1 \pm \sqrt{11}i}{2}$ . Thus the roots of  $f(x)$  are

$$1 \quad \text{(6 points)}, \quad \frac{-1 + \sqrt{11}i}{2} \quad \text{(2 points)}, \quad \frac{-1 - \sqrt{11}i}{2} \quad \text{(2 points)}.$$

As a product of linear factors,

$$f(x) = (x - 1) \left( x - \frac{-1 + \sqrt{11}i}{2} \right) \left( x - \frac{-1 - \sqrt{11}i}{2} \right) \quad \text{(6 points)}.$$

**Problem 3.** The height function is given by  $h(t) = -16t^2 + 10t + 500$  feet.

(a) Rewriting  $h(t)$  as

$$-16t^2 + 10t + 500 = -16\left(t^2 - \frac{10}{16}t\right) + 500 = -16\left(t - \frac{10}{32}\right)^2 + (500 + 16\left(\frac{10}{32}\right)^2)$$

we see that the maximal altitude is  $\approx 501.563$  feet (10 points).

(b) We want to solve  $h(t) = 0$ , where  $t \geq 0$ , or  $-16t^2 + 10t + 500 = 0$ , where  $t \geq 0$ .

By the quadratic formula  $t = \frac{-10 \pm \sqrt{10^2 - (4)(-16)(500)}}{2(-16)}$ , so  $t \approx 5.911$  seconds. (10 points).

**Problem 4.** (a) The calculation (4 points)

$$2 \sin^2(x) + \cos(x) = 2,$$

$$\begin{aligned} 2(1 - \cos^2(x)) + \cos(x) &= 2, \\ -2\cos^2(x) + \cos(x) &= 0, \\ \cos(x)(-2\cos(x) + 1) &= 0, \end{aligned}$$

shows that the set of solutions to the original equation is the set of solutions to  $\cos(x) = 0$  together with the set of solutions to  $\cos(x) = \frac{1}{2}$ . Thus *all* solutions the original equation are described by

$$x = \frac{\pi}{2} + \pi n \text{ (2 points)}, \quad \frac{\pi}{3} + 2\pi n \text{ (2 points)}, \quad \text{or} \quad \frac{5\pi}{3} + 2\pi n \text{ (2 points)},$$

where  $n$  is any integer.

(b) The calculation **(6 points)**

$$\begin{aligned} \log \sqrt{x^2 - 10} &= 3, \\ \sqrt{x^2 - 10} &= 10^3, \\ x^2 - 10 &= (10^3)^2, \\ x^2 &= 10^6 + 10, \\ x &= \pm \sqrt{10^6 + 10}, \end{aligned}$$

the steps of which are reversible, shows that  $x = \sqrt{10^6 + 10}$  **(2 points)**,  $-\sqrt{10^6 + 10}$  **(2 points)**.

**Problem 5.** (a)  $600 \left(1 + \frac{0.08}{4}\right)^{4 \cdot 9}$  **(7 points)**  $\approx 1044.615$  dollars **(3 points)**.

(b) Let  $P$  be the initial amount invested at  $100r\%$  per year compounded continuously. Then  $2P = P(15) = Pe^{r15}$  which means  $2 = e^{r15}$  **(3 points)** since  $P > 0$ . Therefore  $r = \frac{\ln 2}{15}$  **(4 points)**  $\approx .046$  **(3 points)** which can be expressed as 4.6%.

**Problem 6.** (a) All real numbers except for  $x = 9/17$  **(3 points)**. (b)  $y = 5/17$  **(3 points)**. (c)  $x = -9/17$  **(3 points)**. (d) At  $x = -3/5$  **(3 points)**. (e) At  $y = -1/3$  **(3 points)**. (f) Labeling **(3 points)** and shape of graph **(2 points)**.

**Problem 7.** (a) *no* **(2 points)**. Take  $x = \frac{\pi}{4}$  for example. In this case  $\cot(x + \pi) = \cot\left(\frac{\pi}{4} + \pi\right) = 1$  and  $-\cot(x) = -\cot\left(\frac{\pi}{4}\right) = -1$  **(2 points)**.

(b) *no* **(2 points)**. Take  $x = 0$  for example. In this case  $e^{x+5} = e^5$  and  $e^x + e^5 = e^0 + e^5 = 1 + e^5$  **(2 points)**.

(c) *yes* **(4 points)**.

(d) *no* **(2 points)**. Take  $x = 1$  for example. In this case  $\ln(x + 10) = \ln(11) \neq 0$  and  $(\ln 1)(\ln 10) = (0)\ln(10) = 0$  **(2 points)**.

(e) *no* **(2 points)**. Take  $x = 2\pi$  for example. In this case  $\arccos(\cos(2\pi)) = \arccos(\cos(0)) = 0$ , since  $0 \leq x \leq \pi$ , which is not  $x = 2\pi$  **(2 points)**.

**Problem 8.** Since  $\tan(t) = -\frac{7}{6}$  and  $\pi < t < 2\pi$ , it follows that the terminal side of  $t$  is in the fourth quadrant. Therefore  $(6, -7)$  is a point on the terminal side of the angle  $t$ . Since  $r = \sqrt{6^2 + (-7)^2} = \sqrt{85}$  we conclude that

$$\cos(t) = \frac{6}{\sqrt{85}}, \quad \sin(t) = -\frac{7}{\sqrt{85}}, \quad \tan(t) = \frac{\sin(t)}{\cos(t)} = -\frac{7}{6}.$$

The last calculation is not necessary, of course, but does provide a consistency check. Thus

$$(a) \cos(t) = \frac{6}{\sqrt{85}} \text{ (5 points)}, \quad (b) \csc(t) = -\frac{\sqrt{85}}{7} \text{ (5 points)},$$

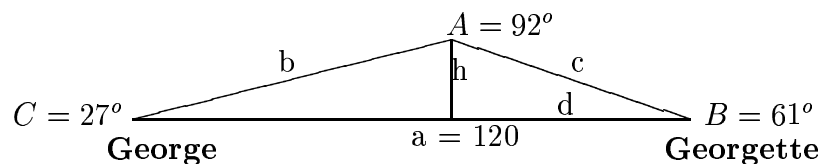
and since  $\frac{\pi}{2} < \frac{t}{2} < \pi$ ,

$$(c) \sin(t/2) = \sqrt{\frac{1 - \cos(t)}{2}} = \sqrt{\frac{1 - \frac{6}{\sqrt{85}}}{2}} = \sqrt{\frac{\sqrt{85} - 6}{2\sqrt{85}}} \text{ (5 points)},$$

and finally

$$(d) \cos(2t) = \cos^2(t) - \sin^2(t) = \left(\frac{6}{\sqrt{85}}\right)^2 - \left(\frac{-7}{\sqrt{85}}\right)^2 = \frac{36}{85} - \frac{49}{85} = -\frac{13}{85} \text{ (5 points)}.$$

**Problem 9.** (a) Let  $A$  be the angle formed by the line segments from George to the top of the streetlight to Georgette, let  $B = 61^\circ$ , and let  $C = 27^\circ$ . Then  $A = 180^\circ - 27^\circ - 61^\circ = 92^\circ$ . Below is a picture *which is not meant to be to scale*.



By the Law of Sines

$$\frac{120}{\sin 92^\circ} = \frac{b}{\sin 61^\circ}.$$

Therefore

$$b = \left(\frac{120}{\sin 92^\circ}\right) \sin 61^\circ \approx 105.018 \text{ feet (4 points)}.$$

Consequently the height of the streetlight is  $h = b \sin 27^\circ \approx 47.677 \text{ feet (8 points)}$ .

(b) The distance from Georgette to the streetlight is

$$d = h \cot 61^\circ = \frac{h}{\tan 61^\circ} \approx 26.428 \text{ feet (8 points)}.$$

**Problem 10.** (a) The modulus of  $1 + \sqrt{3}i$  is  $|1 + \sqrt{3}i| = \sqrt{1^2 + (\sqrt{3})^2} = 2$ . Therefore

$$1 + \sqrt{3}i = 2 \left( \frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = 2 \left( \cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right) \text{ (10 points)}.$$

(b) Using the polar form of  $1 + i$  we calculate

$$\begin{aligned} (1 + \sqrt{3}i)^7 &= \left( 2 \left( \cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right) \right)^7 \\ &= 2^7 \left( \cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right)^7 \\ &= 2^7 \left( \cos\left(\frac{7\pi}{3}\right) + i \sin\left(\frac{7\pi}{3}\right) \right) \\ &= 128 \left( \cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right) \text{ (10 points)}. \end{aligned}$$

Note that  $(1 + \sqrt{3}i)^7 = 64(1 + \sqrt{3}i)$ .