Hour Exam I

Name (print) _____

(1) There are *four questions* on this exam. (2) *Return* this exam copy with your test booklet. (3) *You are expected to abide by the University's rules concerning academic honesty.* (4) *Base the proofs* for problems 2c) and 4 on the axioms for the real number system and:

  a) the product of two positive real numbers is positive,

  b) the product of two negative real numbers is positive,

  c) the product of a positive real number and a negative real number is negative,

  d) the product of zero and any real number is zero, and

  e) if a, b, c are real numbers and $a < b$ then $a - c < b - c$ and $c - a > c - b$.

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1. (25 pts.)   Let P and Q be statements.

  a) Write out the truth table for the statement "(P implies Q) implies P". Include a column for "P implies Q" in your table.

  b) Write out the truth table for the statement "P implies (Q implies P)". Include a column for "Q implies P" in your table.

  c) Are the statements of part a) and part b) logically equivalent? Justify your answer in terms of the truth tables of parts a) and b).

  d) Does the statement of part a) imply the statement of part b)? Justify your answer in terms of the truth tables of parts a) and b).

2. (30 pts.)   Consider the statement "$(a - 4)(a - 7) \geq 0$ implies $a \leq 4$ or $7 \leq a$".

  a) What is the converse of the statement?

  b) What is the contrapositive of the statement? Write it without "not".

  c) Prove the statement by contradiction.

3. (25 pts.)   Prove by induction that the sum of the odd integers

$$1 + 3 + 5 + \cdots + (2n + 1) = (n + 1)^2$$

for all $n \geq 2$.

4. (20 pts.)   Show directly that $a \leq 5$ or $8 < a$ implies $(a - 5)(a - 8) \geq 0$.