Math 215 **Summer 2008** **Radford**

Written Homework # 2

Due at the beginning of class 06/27/08

You may assume the propositions and theorems in the text. You may assume that the product of two real numbers is positive, if both are positive or both are negative, and the product is negative if one is positive and one is negative.

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1. Let n be an integer consider the assertion: If $0 < n < 3$ then $n^3 < -2n^2 + 15n$.

   (a) Prove the assertion by cases.

   (b) Prove the assertion by "working backwards".

   (c) Determine the integers n for which $n^3 < -2n^2 + 15n$, or equivalently $n^3 + 2n^2 - 15n < 0$.

2. Let a be a real number and consider the assertion $a^2 \geq 7a$ implies $a \leq 0$ or $a \geq 7$.

   (a) Prove the assertion by contradiction.

   (b) Prove the assertion directly.

   (c) State the contrapositive of the assertion.

3. Let a be a real number and consider the assertion $a^2 - 12a + 35 < 0$ implies $5 \leq a < 7$.

   (a) Prove the assertion by contradiction.

(b) Prove the assertion directly.

(c) Is the converse true? Give a proof or find a counterexample.

4. An integer n is *even* if $n = 2m$ for some integer m and is *odd* if $n = 2m+1$ for some integer m.

(a) Prove, by cases, that the sum of two integers is even, if both are even or both are odd, and the sum is odd if one is even and the other is odd.

(b) Prove, by cases, that $n^2 + 5n$ is even for all integers n.

5. Prove, by induction, that $n^2 + 5n$ is even for all integers $n \geq 1$.