Math 215 **Summer 2008** **Radford**

Written Homework # 4

Due at the beginning of class 07/11/08

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1. We define the intersection of sets $A_1, \ldots, , A_n$ inductively by

$$A_1 \cap \cdots \cap A_n = \begin{cases} A_1 & : \ n = 1; \\ (A_1 \cap \cdots \cap A_{n-1}) \cap A_n & : \ n > 1. \end{cases}$$

(a) Suppose that A, B, and C are sets and $C \subseteq A, B$. Show that $C \subseteq A \cap B$.

(b) Suppose that $C, A_1, \ldots, A_n$ are sets and $C \subseteq A_1, \ldots, A_n$. Construct a proof by induction that $C \subseteq A_1 \cap \cdots \cap A_n$.

2. Describe the power set of S in the following cases by listing its elements:

(a) $S = \emptyset$; (b) $S = \{7\}$; (c) $S = \{\emptyset\}$; (d) $S = \{6, 9\}$.

3. Let A, B, and C be sets. Complete the following truth table

$x \in A$	$x \in B$	$x \in C$	$x \in A \cup B$	$x \in (A \cup B) \cup C$	$x \in B \cup C$	$x \in A \cup (B \cup C)$

and use it to explain why $(A \cup B) \cup C = A \cup (B \cup C)$.

4. Let U be a universal set and $A, B \subseteq U$. Complete the following truth table

$x \in A$	$x \in B$	$x \in A \cap B$	$x \in (A \cap B)^c$	$x \in A^c$	$x \in B^c$	$x \in A^c \cup B^c$

and use it to explain why $(A \cap B)^c = A^c \cup B^c$, one of De Morgan's Laws.

5. Let U be a universal set and $A, B \subseteq U$. Using the fact that $A^{c\,c} = A$, show that $(A \cap B)^c = A^c \cup B^c$ implies $(A \cup B)^c = A^c \cap B^c$, De Morgan's other law.