

Name (print) _____ Tu/Th Discussion (circle) 12 1 2

***** If you use a calculator it must be your own. You must show your work. *****

1. (12 points) Find the *exact value* ofa) $\sin\left(\frac{7\pi}{12}\right)$ and

$$\begin{aligned} \text{Solution: } \sin\left(\frac{7\pi}{12}\right) &= \sin\left(\frac{1}{2}\left(\frac{7\pi}{6}\right)\right) = \sqrt{\frac{1 - \cos\left(\frac{7\pi}{6}\right)}{2}} = \sqrt{\frac{1 - (-\cos\left(\frac{\pi}{6}\right))}{2}} \\ &= \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2} \quad (\mathbf{6 \text{ points}}) \text{ using the half-angle formula.} \end{aligned}$$

b) $\cos\left(\frac{7\pi}{12}\right)$.

$$\begin{aligned} \text{Solution: } \cos\left(\frac{7\pi}{12}\right) &= \cos\left(\frac{1}{2}\left(\frac{7\pi}{6}\right)\right) = -\sqrt{\frac{1 + \cos\left(\frac{7\pi}{6}\right)}{2}} = -\sqrt{\frac{1 + (-\cos\left(\frac{\pi}{6}\right))}{2}} \\ &= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\frac{\sqrt{2 - \sqrt{3}}}{2} \quad (\mathbf{6 \text{ points}}) \text{ using the half-angle formula.} \end{aligned}$$

2. (8 points) Find the *exact value* of *all* solutions to $\sin t = -\frac{\sqrt{3}}{2}$.

Solution: The solutions $0 \leq t < 2\pi$ are $t = \frac{4\pi}{3}, \frac{5\pi}{3}$ (**3 points**). Thus all solutions are described by

$$\frac{4\pi}{3} + 2\pi n, \quad \text{or} \quad \frac{5\pi}{3} + 2\pi n,$$

where n is an integer (**5 points**).