Math 121 F11 Lowman

Special Assignment #2

Solutions for problems 6 thru end.

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6. $f(x) = \dfrac{\sqrt{x-1}}{x-3}$

The numerator can only use real #s where $x - 1 \geq 0$ Solve for x.

$$\begin{aligned} x - 1 &\geq 0 \\ +1 \quad & \quad +1 \\ x &\geq 1 \end{aligned}$$

can be used in numerator

The denominator cannot be zero.

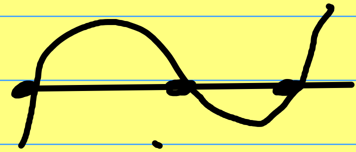
$x - 3 \neq 0 \Rightarrow x \neq 3$

<u>Domain</u> $D = \{ x \in \mathbb{R} \mid x \geq 0 , \; x \neq 3 \}$

means <u>and</u>

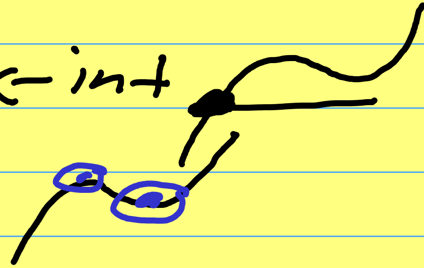
7. $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$

(a) degree is odd



(b) deg $= 3 \Rightarrow$ at most $\underline{3}$ x-ints (i.e. real roots)

(c) odd deg $\Rightarrow$ at least one x-int

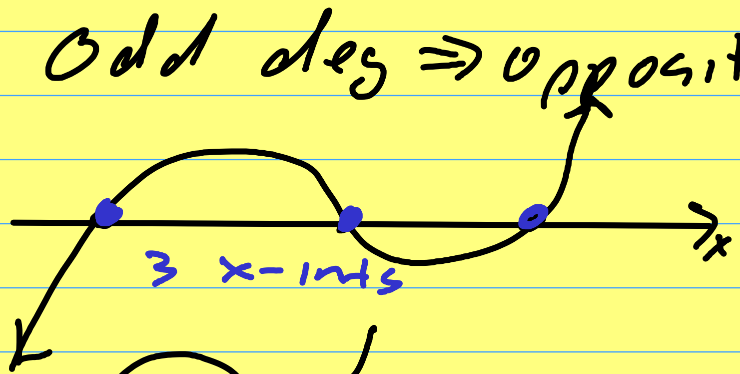


(d) at most 2 local max/min.

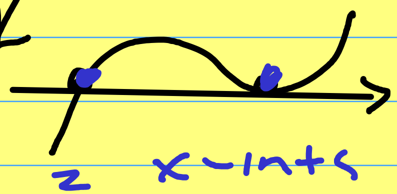
(e) Polynomials are smooth continuous functions define for all real numbers.
$\Rightarrow$ all polynomials have one y-intercept.

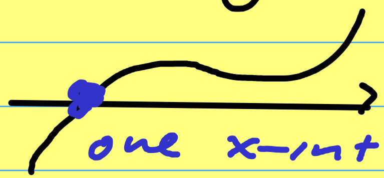
(f) Odd deg $\Rightarrow$ opposite ends go in opposite directions

(g)

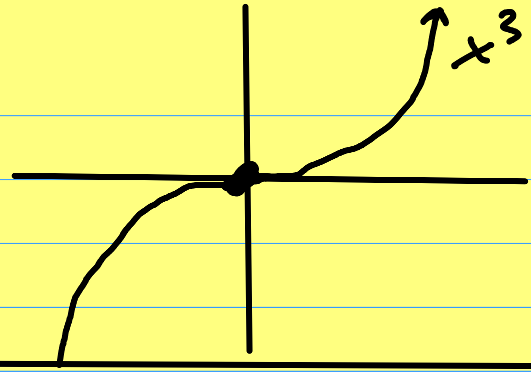


General Shape of 3rd degree polynomial





Special case $f(x) = x^3$



(h) degree 4

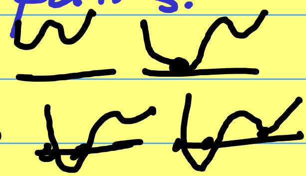
$$f(x) = ax^4 + bx^3 + cx^2 + dx + e$$

(a) deg even

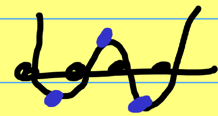
(b) deg = 4 $\Rightarrow$ at most 4 x-ints (ie real roots)

Note: will always have 4 roots if can be repeated or Complex. Complex roots always come in pairs.

(c) deg even $\Rightarrow$ at most 0 x-ints

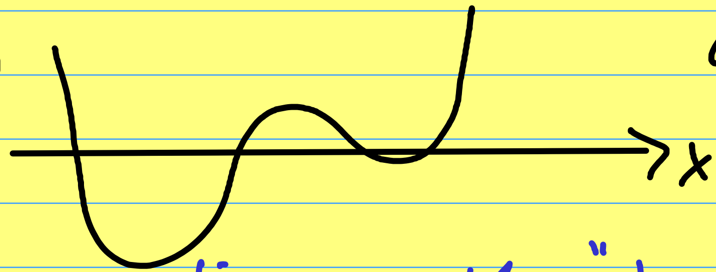


(d) deg = 4 $\Rightarrow$ at most (deg−1) = (4−1) = 3 local max/min points.



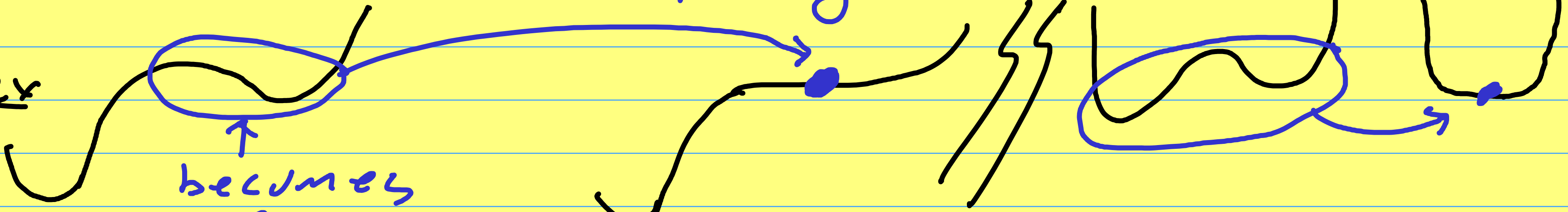
(e) All polynomials have one y-int.

(f) even degree $\Rightarrow$ Both ends go up or both ends go down.

(g) 

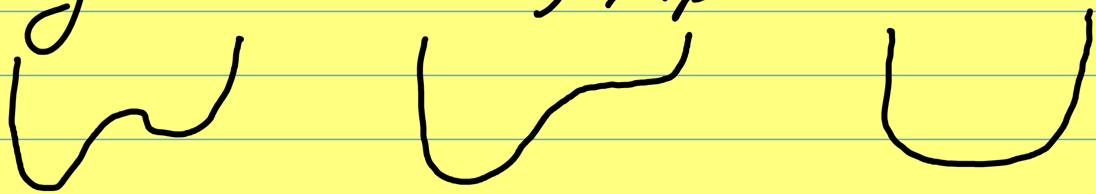
General shape of 4th deg polynomial.

Sometimes the "bumps" get smoothed out.

ex 

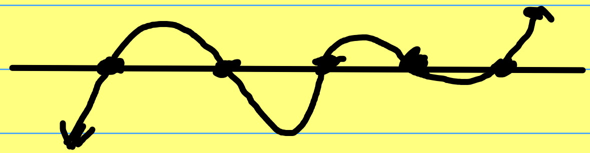
becomes one point

This gives different variations of the general shape.

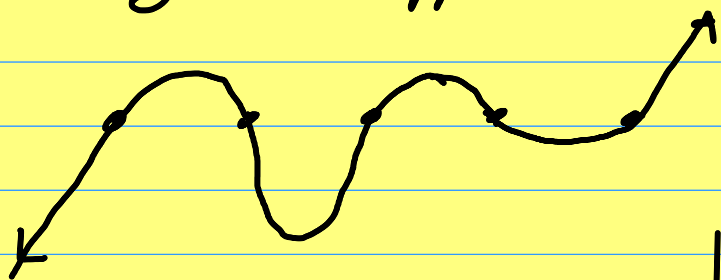


7. $f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + g$

(a) deg odd

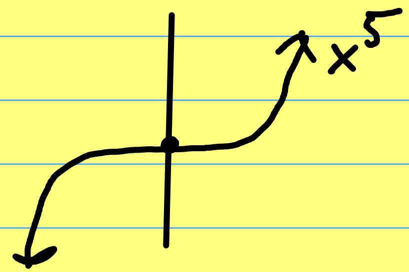


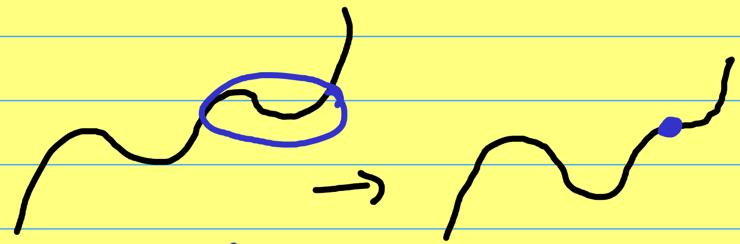
(b) at most 5 x-ints
(c) at least one x-int
(d.) at most $(5-1) = 4$ local max/min points
(e) one y-int
(f) odd deg ⇒ opposite directions on ends.
(g)



general shape

special cases:





8. $f(x) = (\underbrace{2x^3 - 9x + 13}_{g(x)})^5$

let $g(x) = 2x^3 - 9x + 13$

and $h(y) = y^5$ $\quad$ (or $h(x) = x^5$)

Then $f(x) = h(g(x)) = (g(x))^5 = (2x^3 - 9x + 13)^5$

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9. $y = f(x) = x^2 + x$

T_1: shift Right by 3 $\Rightarrow$ $x \to (x-3)$

$y_1 = (x-3)^2 + (x-3)$

T_2: Stretch vertically by factor 4 $\Rightarrow$ $y_1 \to \frac{y_2}{4}$

$\frac{y_2}{4} = (x-3)^2 + (x-3) \to y_2 = 4[(x-3)^2 + (x-3)]$

T_3: Shift down by 5 $\Rightarrow y_2 \to y_3 - (-5) = y_3 + 5$

$$y_3 + 5 = 4\left[(x-3)^2 + (x-3)\right]$$

$$y_3 = 4\left[(x-3)^2 + (x-3)\right] - 5 \quad \checkmark$$

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10. $x - 3 = -\sqrt{5-x}$

$$(x-3)^2 = \left(-\sqrt{5-x}\right)^2$$

squaring both sides $\Rightarrow$ Need to check answers at end.

$$x^2 - 6x + 9 = \left(\sqrt{5-x}\right)^2$$

$$x^2 - 6x + 9 = 5 - x$$

$$+x \quad -5 \quad = \quad -5 \quad +x$$

$$x^2 - 5x + 4 = 0$$

$$(\underbrace{x - 4}_{=0})(\underbrace{x - 1}_{=0}) = 0$$

must check these.

$x = 4$, $x = 1$ are possible solutions

Check if $x=4$ and $x=1$ are Solutions by plugging into the Original Equation

Try $x=4$:
$$4-3 \stackrel{?}{=} -\sqrt{5-4}$$
$$1 = -\sqrt{1}$$
$$1 = -\sqrt{1} \Rightarrow \text{No Bueno.}$$

$x=4$ is **not** a solution

Try $x=1$:
$$1-3 \stackrel{?}{=} -\sqrt{5-1}$$
$$-2 = -\sqrt{4}$$
$$-2 = -2 \Rightarrow \text{Muy Bueno}$$

$x=1$ is the only solution.

11. $2x^2 + 8x - 16 = 0$ Solve by Completing the Square.

$$\frac{1}{2}(2x^2 + 8x - 16) = \frac{1}{2}(0)$$

$$\cancel{\tfrac{1}{2}\cdot 2}^{1}x^2 + \cancel{\tfrac{1}{2}8}^{4}x - \cancel{\tfrac{1}{2}\cdot 16}^{8} = 0$$

$$x^2 + 4x - 8 = 0$$

$$+8 = +8$$

$$x^2 + 4x + \quad = 8$$

Need half of this squared here

$$\left(\frac{1}{2}\cdot 4\right)^2 = 2^2 = 4 \implies \text{add 4 to both sides.}$$

$$x^2 + 4x + 4 = 8 + 4$$

$$(x+2)^2 = 12$$

$$\sqrt{(x+2)^2} = \pm\sqrt{12}$$

↑ so don't lose a root.

$$x + 2 = \pm\sqrt{12}$$

note $\sqrt{12} = \sqrt{4\cdot 3} = \sqrt{4}\cdot\sqrt{3}$
$= 2\cdot\sqrt{3}$

$$x + 2 = \pm 2\sqrt{3}$$
$$-2 \quad = -2$$

$$x = -2 \pm 2\sqrt{3}$$

$x_1 = -2 + 2\sqrt{3}$

$x_2 = -2 - 2\sqrt{3}$

Now find x_v the x-coord of the vertex.

$$x_v = \frac{x_1 + x_2}{2}$$

cancel

$$= \frac{(-2 + \cancel{2\sqrt{3}}) + (-2 - \cancel{2\sqrt{3}})}{2}$$

$$= \frac{(-2) + (-2)}{2} = \frac{\cancel{2}(-2)}{\cancel{2}} = -2$$

$$\boxed{x_v = -2}$$

11. $ax^2 + bx + c = 0$

Solve Quadratic Equation by Completing Square

$$\frac{1}{a}(ax^2 + bx + c) = \frac{1}{2}(0)$$

$$\frac{1}{\cancel{a}} \cdot \cancel{a}x^2 + \frac{1}{a} \cdot bx + \frac{1}{a} \cdot c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$-\frac{c}{a} = \frac{-c}{a}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Need half of this squared here

$$\frac{1}{2}\left(\frac{b}{a}\right) \longrightarrow \left(\frac{b}{2a}\right)^2 \Rightarrow \text{add } \left(\frac{b^2}{2a}\right)^2 \text{ to both sides}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right)$$

$=1$ but over common denominator

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$-\frac{b}{2a} = -\frac{b}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

or

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

$$x_1 = \frac{-b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}$$

$$x_2 = \frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}$$

x coordinate of vertex is

$$x_v = \frac{x_1 + x_2}{2}$$

cancel

$$= \frac{\left(\frac{-b}{2a} + \cancel{\frac{\sqrt{b^2-4ac}}{2a}}\right) + \left(\frac{-b}{2a} - \cancel{\frac{\sqrt{b^2-4ac}}{2a}}\right)}{2}$$

$$= \frac{\left(\frac{-b}{2a}\right) + \left(\frac{-b}{2a}\right)}{2} = \frac{\overset{1}{\cancel{2}}\left(\frac{-b}{2a}\right)}{\cancel{2}}$$

$$\boxed{x_v = \frac{-b}{2a}}$$