

Math 121 Special Assignment I Lowman Fall 2011

No Calculators. In all problems you must show your work and your answers must be clearly labeled. If the grader has to guess which part of what you wrote down is actually the answer then you will not receive credit. When possible put a box around your answer and clearly label it. All work must be written in the exam booklet. Put your name, your TA's name and your UIN on the exam booklet. Do not write in the upper right corner of the booklet, this is used to write your scores. You can keep the exam sheet but you must turn in the exam booklet. If no work then no credit.

Note, these will be the actual instructions when you take your exam. For the special assignment, you do not need to use an exam booklet.

1. Given that $3i$ is a root of $f(x) = x^4 + 10x^2 + 9$, find all remaining roots (real and complex) and then **write the function as a product of four linear factors**. You must use long or synthetic division and the fact that complex roots will occur in conjugate pairs. You will not get credit if you use other methods or do not show all steps.
2. Write the rule of a function $g(x)$ obtained by performing the following transformations (one after the other) on $f(x) = x^4$. Your final answer should be in the form $y = g(x)$. Show your work for each step.
 - (a) shift up by 5.
 - (b) reflect about the x axis
 - (c) shift horizontally 3 units to the right,
 - (d) reflect about the y axis.
 - (e) stretch vertically by a factor of 4,
3. Find the inverse of the function $f(x) = \frac{3(4-x)}{x-2}$
4. Given $f(x) = \frac{3(4-x)}{x-2}$, find the intervals where $f(x) > 0$ and $f(x) < 0$
5. Consider the rational function $f(x) = \frac{3(4-x)}{x-2}$:
 - (a) Find all x-intercepts and y-intercepts.
 - (b) Find the vertical asymptote. Determine what is happening to the left of the VA by plugging in a number to the left of the VA. Repeat to the right.
 - (c) Determine the Right end behavior of $f(x)$ i.e.
as $x \rightarrow \infty$, $f(x) \rightarrow ?$ You should find a Horizontal asymptote to the right. Give the equation of the asymptote and determine if $f(x)$ is approaching from above or below by plugging a number into $f(x)$ and comparing to the HA line
 - (d) Determine the Left end behavior of $f(x)$ i.e.
as $x \rightarrow -\infty$, $f(x) \rightarrow ?$ You should find a Horizontal asymptote to the left. Give the equation of the asymptote and determine if $f(x)$ is approaching from above or below by plugging a number into $f(x)$ and comparing to the HA line

- (e) Using the above information, sketch a graph of $f(x)$ labeling all of the above on the graph. Use at least **1/2** page for your graph.
6. Find the domain and range of the function: $f(x) = \frac{\sqrt{x-1}}{x-3}$
7. Given the general 3rd degree polynomial $f(x) = ax^3 + bx^2 + cx + d$
- Is the degree even or odd?
 - $f(x)$ has at most how many x-intercepts?
 - $f(x)$ has at least how many x-intercepts?
 - $f(x)$ has at most how many local max/min points?
 - $f(x)$ has how many y-intercepts?
 - Do the right and left ends of the graph of $f(x)$ go in the same directions or opposite directions?
 - Draw a graph of a general 3rd degree polynomial. In addition, draw graphs for a few special cases.
 - Repeat (a) through (g) for a general 4th degree polynomial.
 - Repeat (a) through (g) for a general 5th degree polynomial.
8. Write a function $f(x) = (2x^3 - 9x + 13)^5$ as a composite of two functions, neither of which is the identity function.
9. Describe a sequence of transformations that will transform the graph of $f(x) = x^2 + x$ into the graph of the function $g(x) = 4((x - 3)^2 + (x - 3)) - 5$
10. Find a solution to the equation: $x - 3 = -\sqrt{5 - x}$. Show all work.
11. Solve $2x^2 + 8x - 16 = 0$ by first completing the square. Find the x-coord of the vertex. Show all work.
12. Solve $ax^2 + bx + c = 0$ by completing the square. Find the x-coord of the vertex. Show all work.