Math 121 Final Exam Review Fall 2011

- **Calculators can be used. No Cell Phones. Your cell phones cannot be used for a calculator**.
- PUT YOUR NAME, UIN, INSTRUCTORS NAME, TA'S NAME AND DISCUSSION TIME ON THE EXAM BOOKLET. DO NOT WRITE IN THE UPPER RIGHT CORNER OF THE BOOKLET, THIS IS USED FOR GRADING. ALSO INCLUDE MATH 121, FINAL EXAM, FALL 2011.
- SHOW ALL WORK IN EXAM BOOKLET. CLEARLY LABEL AND BOX ANSWERS. IF NO WORK THEN NO CREDIT.
- PLACE YOUR EXAM SHEET INSIDE YOUR EXAM BOOKLET, THEN TURN IN BOTH YOUR EXAM SHEET AND BOOKLET. YOU MUST FINISH BY 8:00PM.
- YOU MUST SIGN YOUR EXAM BOOKLET NEXT TO YOUR NAME.

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1. Given that $2i$ is a root of $f(x) = x^4 - 5x^3 + 10x^2 - 20x + 24$, find all remaining roots (real and complex) and then **write the function as a product of four linear factors**. **You must use** long or synthetic division and the fact that complex roots will occur in conjugate pairs. You will not get credit if you use other methods or do not show all all steps.

2. Write the rule of a function $g(x)$ obtained by performing the following transformations (one after the other) on $f(x) = \frac{1}{x}$. Your final answer should be in the form $y = g(x)$. Show your work for each step.

   (a) T_1 **:** shift **left** by 1.
   (b) T_2 **:** stretch horozontally by 2.
   (c) T_3 **:** stretch vertically by 3.
   (d) T_4 **:** shift up by 4.
   (e) T_5 **:** reflect about the y axis.

3. Find the inverse of the function $f(x) = \frac{1+x}{2x+3}$

4. Given $f(x) = \frac{2(x+3)}{x-2}$, find the intervals where $f(x) > 0$ and $f(x) < 0$ You must show your work.

5. Consider the rational function $f(x) = \frac{2(x+3)}{x-2}$:

   (a) Find the domain and range of the function: $f(x)$
   (b) Find all x-intercepts and y-intercepts.
   (c) Find the vertical asymptote. Determine what is happening to the left of the VA by plugging in a number to the left of the VA. Repeat to the right.
   (d) Determine the Right end behavior of $f(x)$ i.e. as $x \rightarrow \infty,\quad f(x) \rightarrow ?$ You should find a Horizontal asymptote to the right. Give the equation of the asymptote and determine if $f(x)$ is aproaching from above or below by plugging a number into $f(x)$ and comparing to the HA line.
   (e) Determine the Left end behavior of $f(x)$ i.e. as $x \rightarrow -\infty,\quad f(x) \rightarrow ?$ You should find a Horizontal asymptote to the left. Give the equation of the asymptote and determine if $f(x)$ is aproaching from above or below by plugging a number into $f(x)$ and comparing to the HA line.
   (f) Using the above information, sketch a graph of $f(x)$ labeling all of the above on the graph. Use at least $1/2$ page for your graph.
   (g) Is your answer for problem 4 consistent with your graph? Why or why not. Give the correct answer based on your graph and your answer for problem 4.

6. (15 points) Let $f(x) = 3x^2 - x$. Evaluate the difference quotient

$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

7. (20 points) Solve the inequality, and express the solution using interval notation.

$$\frac{3x-5}{x+2} \leq 2$$

8. (10 points) Find the function that is finally graphed after each of the following transformations is applied to the graph of $f(x) = \sqrt{x}$.
   (a) Reflect about the y-axis
   (b) Shift left 2 units
   (c) Shift down 3 units

9. (15 points) Find the vertical asymptote(s), horizontal/oblique asymptote(s), if any, of the given function

$$G(x) = \frac{6x^2 + 7x - 5}{3x + 5}$$

10. (20 points) Use the given zero $1 + 3i$ to find the remaining zeros of the function $f(x) = x^4 - 7x^3 + 14x^2 - 38x - 60$.

11. (20 points) Given

$$f(x) = \frac{x^2 + 3x - 10}{x^2 + 8x + 15}$$

   (a) Find the domain of $f(x)$ and the y-intercept.

   (b) Find the x-intercept(s), and determine the behavior of the graph of $f(x)$ near each x-intercept.

   $\rightarrow$ turn over

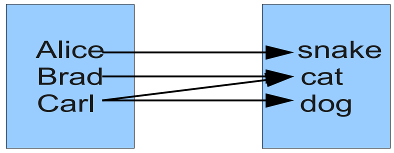
   (c) Locate the vertical asymptote(s) and any horizontal/oblique asymptote(s) of the graph. Check whether the graph of $f(x)$ intersects the horizontal/oblique asymptote(s).

   (d) Using the real zeros of the numerator and denominator of $f(x)$, divide the x-axis into intervals and determine where the graph is above the x-axis and where it is below the x-axis by choosing a number in each interval and evaluating $f(x)$.

   (e) Put all the information together to obtain the graph of $f(x)$.

12. Use the Rational Zeros Theorem to find all the real zeros of $f(x) = x^3 + 2x^2 - 5x - 6$. Write $f(x)$ in factored form.

13. Determine whether the relation rerpresents a function. If it is a function, state the domain and range.



14. . Determine the average rate of change for the function $f(x) = \frac{3}{4}x + 3$

15. Determine, without graphing, whether the quadratic function $f(x) = x^2 + 2x - 2$ has a maximum value or a minimum value and them find that value.

16. Graph the function.

$$f(x) = \begin{cases} -x+2 & x < 0 \\ \sqrt{x}+3 & x \geq 0 \end{cases}$$

17. Give the equation of the horizontal asymptote, if any, of the function. $f(x) = \dfrac{x^2 - 2}{4x - x^4}$

18. Find the vertical asymptotes of the rational function. $f(x) = \dfrac{x - 4}{16x - x^3}$

19. Solve the inequality. $x + \dfrac{18}{x} < 9$

20. List the potential rational zeros of the polynomial function. Do not find the zeros. $f(x) = 5x^4 - x^2 + 3$

21. Information is given about a polynomial $f(x)$ whose coefficients are real numbers. Find the remaining zeros of f. Degree 6; zeros: $-6, 4, 6-5i, -4+i$

22. The function f is one-to-one. Find the inverse. $f(x) = \dfrac{4}{x+4}$

23. Solve the equation in the real number system. $2x^3 - x^2 - 14x + 7 = 0$

24. Given $f(x) = 4x - 6x^2$, find a) f(-3) 4pts b) $\dfrac{f(x+h) - f(x)}{h}$ (answer in simplified form!!!) 8pts

25. Given the function $R(x) = \dfrac{x+4}{5-2x}$, answer the following: 16pts

    a) Show whether $\left(\dfrac{3}{2}, \dfrac{11}{2}\right)$ is a point on the graph of R(x).

    b) Find ALL intercepts, in proper form.

    c) If $f(x) = 2$, what does x equal? (Show Work)

26. For the Quadratic function $\boldsymbol{f(x) = -2x^2 + 5x + 3}$, find: 16pts
a) the Vertex and Axis of Symmetry
b) ALL intercepts, in proper form.
c) Sketch a graph, with coordinates shown for values found above.

27. Solve the Inequality, putting answer into Interval Notation: $\boldsymbol{x^4 \geq 9x^2}$ 12pts

28. A rational function is given below. Find the requested information and SKETCH a rough graph of $\boldsymbol{R(x) = \dfrac{2x^2 = 8}{x^2 - 5x + 6}}$ 16pts a) List ALL Intercepts, in proper form. Also, name the coordinates of any holes in the graph. b) List ALL Asymptotes(Vert, Hor, Obl), in proper form. c) Sketch a graph with the above information, plus several coordinate pairs labeled correctly.

29. Perform the following and find ALL COMPLEX roots of $\boldsymbol{f(x) = 2x^4 + 7x^3 - 24x^2 + 34 - 12}$ 18pts a) List ALL POSSIBLE Rational Zeros b) Using Synthetic Division and/or the fact that $\boldsymbol{x = 1i}$ is a Root, find the remaining roots of $\boldsymbol{f(x)}$.

30. Find the rule of the function which resembles $\boldsymbol{y = \sqrt(x)}$ but has been shifted down 7, shifted left 3, and then reflected over the x-axis. 10pts

31. Fill in all boxes of the table with EXACT values.

$\boldsymbol{\theta}$ degrees	$\boldsymbol{\theta}$ radians	$\boldsymbol{\sin(\theta)}$	$\boldsymbol{\cos(\theta)}$
0			
30			
45			
60			
90			

32. In the boxes complete the trigonometric identities as given in lectures

left side of identity		right side of identity
$\boldsymbol{\sin(x+y)}$	=	
$\boldsymbol{\cos(x+y)}$	=	
$\boldsymbol{\sin(2x)}$	=	
$\boldsymbol{\cos(2x)}$	=	
$\boldsymbol{\cos^2(x)}$in terms of $\boldsymbol{\sin^2(x)}$	=	
half angle identity for $\boldsymbol{\sin^2(x)}$	=	

33. If $\tan(\theta) = \frac{-4}{3}$ and $\cos(\theta) > 0$, find $\sin(\theta)$ and $\cos(\theta)$.

34. Find all solutions to:

$$\frac{log_3(10)}{log_3(e)} + \frac{1}{3} \cdot \log_2(2^3) \cdot e^{ln(3) \cdot x} - ln(10) = 3^{(5-6/x)} \cdot ln(e) + 99 * ln(1) \cdot 10^{(x^2+1)}$$

35. Solve for t when P is two times A:

$$P = \frac{A}{1 - B \cdot 2^{-rt}}$$

Show all steps and box your answer.

36. A wheel with radius $r = 24\ in$ is rolling at a speed of $44\ ft/sec$.

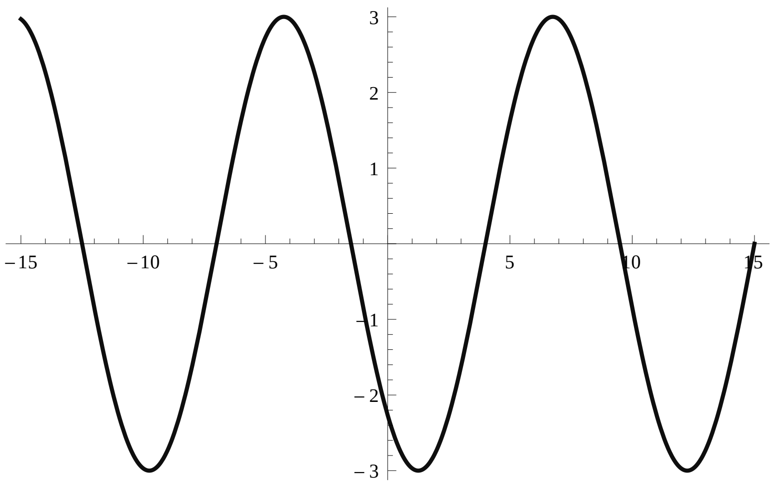
(a) What is ω the ***angular speed***, in **radians per second**?

(b) Convert your answer to **rpms** (rotations per minute).

Show all work, including units, for full credit. Give your answer in terms of π and reduce fractions when possible.
$5280ft = 1mile$, $1in = 2.54cm$. $1km = 1000m$, $1m = 100cm$, $1\ rotation = 2\pi\ radians = 360\ degrees$, $1min = 60sec$.

37. Given $y = A\sin(\omega(x - x_0)) = A\sin(\omega x - \phi)$ Find:

- amplitude $A =$ ________________
- period $T =$ ________________
- angular frequency $\omega = \frac{2\pi}{T} =$ ________________
- phase shift $x_0 =$ ________________
- phase constant $\phi =$ ________________
- phase $\omega x - \phi =$ ________________



38. Let $f(x) = \dfrac{3}{2x+5}$.

(a) (5 pts) Find the inverse of $f(x)$.

(b) (5 pts) Show that $f(f^{-1}(x)) = x$.

(c) (5 pts) State the domain and the range of f.

39. Solve each equation.(Express the solution in an exact form).

(a) (10 pts) $5^{x-2} = 3^{2x+1}$

(b) (10 pts) $\log_2 x^4 - \log_4 x = 7$

40. (a) (7 pts)Find the amount that results from investing \$100 at 6% compounded quarterly after a period of 3 years.

(b) (8 pts) Find the principal (the present value) needed now to get \$300 after 4 years at 8% compounded continuously.

41. A culture of bacteria obeys the law of uninhibited growth.

(a) (5 pts) If N is the number of bacteria in the culture and t is the time in hours, express N as a function of t.

(b) (10 pts) If 500 bacteria are present initially and there are 800 after 1 hour, how many will be present in the culture after 5 hours? Round your answer to two decimal places.

42. (20 pts) Given $\tan\theta = -\dfrac{12}{5}$, $\sin\theta > 0$. Find the exact value of each of the remaining trigonometric functions of θ.

43. Given $f(x) = 2\sin(\frac{\pi}{4}x - \pi)$.

    (a) (5 pts) Find the amplitude, period, and phase shift of $f(x)$.

    (b) (10 pts) Find the five key points of $f(x)$.

44. Solve the equation. $64^{x-4} = 16^{3x}$

45. Use the Change-of-Base Formula and a calculator to evaluate the logarithm. Round your answer to three decimal places. $\log_3 281.9$

46. Solve the equation. $\log(3+x) - \log(x-5) = \log 3$

47. Find the effective rate of interest for 4.25% compounded monthly.

48. Solve the problem. A fossilized leaf contains 6% of its normal amount of carbon 14. How old is the fissil (to the nearest year)? Use 5600 years as the half-life of carbon 14.

49. If A denotes the area of a sector of a circle of radius r formed by the central angle θ, find the missng quantity. If necessary, round the answers to two decimal places. $\theta = 5$ radians, $A = 56$ square meters, $r = ?$

50. In the problem, $\sin\theta$ and $\cos\theta$ are given. Find the exact value of the indicated trigonometric function. $\sin\theta = \frac{1}{4}, \cos\theta = \frac{\sqrt{15}}{4}$ Find $\cot\theta$.

51. Solve the problem. For the equation $y = -\frac{1}{2}\cos(2x - 2\pi)$, identify (i) the amplitude, (ii) the phase shift, and (iii) the period.

52. Write the equation of a sine function that has the given characteristics. Amplitude: 4, Period: 3.

53. Graph the function. $y = \sec\left(\frac{1}{3}x\right)$

54. a) Find the Inverse function to $f(x) = \dfrac{2x+3}{x-3}$: 10pts
    b) Name in INTERVAL NOTATION, the Domain and Range of each, $f(x)$ and $f^{-1}(x)$ 4pts

55. Use properties of logarithms to perform the requested conversion, showing ALL WORK: 8pts each
    a) $\log_5\left(\dfrac{x^4}{y^7\sqrt[6]{z}}\right)$, as two or more logs with NO exponents
    b) $5\ln(2x+3) + 3\ln(x-1) - 4\ln x$, as a single logarithm

56. Solve the equations for all VALID solutions, showing answers in EXACT FORM(No Decimals)
a) $\boldsymbol{4^{x+2} \cdot 2^x = 64}$ b) $\boldsymbol{\log_6(x+2) = 2 - \log_6(7+x)}$ 8pts each

57. The half-life of a newly discovered radioactive isotope is 8.6 years, and it follows the Law of Uninhibited Decay, 15pts
a) Use this information to determine k, rounded to 4 places behind the decimal
A secret government lab accidentally spills 250g. Determine:
b) how much of the 250g is left after 20 years?
c) how many years(to the nearest 0.1) until there is only 25g left?

58. A lawn sprinkler sprays water from its spout all along the way to 32 feet out. The sprinkler cycles through
a turn of $\boldsymbol{195^o}$. How many square feet of lawn is sprinkled? SHOW WORK.
(Hint: possible useful formulas: $\boldsymbol{s = r\theta}$, $\boldsymbol{A = \frac{1}{2}r^2\theta}$) 8pts

59. Given the following information, determine the EXACT VALUE of the remaining 5 Trig functions
Denominators DO NOT need to be rationalized. $\boldsymbol{\csc\theta = \frac{5}{2}}$ and $\boldsymbol{\tan\theta < 0}$ 15pts

60. Given $\boldsymbol{y = -6\sin\left(2x + \frac{3\pi}{2}\right)}$, find: 16pts
a) the Period b) the Amplitude c) the Phase Shift
d) a Sketch of 2 full periods, starting at YOUR PHASE SHIFT, where the x-axis is in RADIANS and all important x-values are CLEARLY marked along the axis.

61. Use the **Law of Sines** to solve the **SAA** Triangle for side $\boldsymbol{b}$ only. Do not solve for side $\boldsymbol{c}$ **:**
$\boldsymbol{A = 40^o, B = 60^o, a = 4}$. Show all work and give answers rounded to one decimal place.

62. Use the **Law of Cosines** to solve the SSS Triangle angle $\boldsymbol{A}$ only. Assume standard triangle notation for angles $\boldsymbol{A, B, C}$ and and sides $\boldsymbol{a, b, c}$:
$\boldsymbol{a = 3, b = 5, c = 6}$. Show all work and give answer in degrees rounded to two decimal places.

63. Given $\boldsymbol{\theta = \tan^{-1}(\frac{1}{2})}$ find $\boldsymbol{\sin\theta}$. Give the exact answer, show your work and do not use a calculator. Hint, draw a right triangle and find the lengths of all sides.

64. Simplify: $\boldsymbol{\dfrac{\sin(2\theta)}{1-\cos(2\theta)}}$. Show all steps. Note the minus sign.

65. Find the exact value of $\boldsymbol{\tan(\frac{5\pi}{6})}$. Show your work and do not use a calculator.

66. Find **All** of the exact solutions to: $\boldsymbol{2\sin(4\theta) = \sqrt{3}}$. Show your work and do not use a calculator.

67. Find the exact value of: $\boldsymbol{\tan^{-1}(\tan\frac{5\pi}{4})}$. Give the exact answer, show your work and do not use a calculator. Hint, first find $\boldsymbol{\tan\frac{5\pi}{4}}$

68. Find all solutions to the following equation on the interval $\boldsymbol{[0, 2\pi)}$:
$\boldsymbol{\sin\theta \cdot \cos\theta - \frac{1}{2}\cos\theta = 0}$. Give the exact answer, show your work and do not use a calculator.

69. (15 points) Write $\boldsymbol{\cos(\sin^{-1} u)}$ as an algebraic expression in $\boldsymbol{u}$.

70. (15 points) Find the exact value of

  (a) $\mathbf{\sin^{-1}(\sin \frac{5\pi}{4})}$.

  (b) $\mathbf{\cos[\cos^{-1}(1.3)]}$

For problem 3, side $\boldsymbol{a}$ is opposite angle A, side b is opposite angle B, side c is opposite angle C. Round all answers to two decimal places.

71. (15 points) Solve the triangle: $\boldsymbol{b = 8,\ c = 9, B = 30^\circ}$

72. (10 points) Find the area of the triangle: $\boldsymbol{a = 8,\ b = 4,\ C = 70^\circ}$

73. (15 points) Establish the given identity.

$$\frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} = 2 \csc x$$

74. (15 points) Let

$$\cos \alpha = \frac{1}{2},\ 0 < \alpha < \frac{\pi}{2}$$
$$\sin \beta = -\frac{4}{5},\ \ -\frac{\pi}{2} < \beta < 0$$

Find the exact solution of $\boldsymbol{\sin(\alpha - \beta)}$.

75. (15 points) Solve the equation.

$$\cos^2 \theta = 4(\sin \theta + 1)$$

Give a general formula of all solutions. Decimal approximations will not be accepted.

76. Find the exact value of the expression. $\boldsymbol{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)}$

77. The displacement $\boldsymbol{d}$ (in meters) of an object in harmonic motion at time $\boldsymbol{t}$ (in seconds) is given. Describe the motion of the object. What is the maximum displacement from its resting position, the time required for one oscillation, and the frequency? $\boldsymbol{d = 3\cos(5t)}$

78. Solve the equation $\boldsymbol{4\sin^2\theta - 3 = 0}$ on the interval $\boldsymbol{0 \le \theta < 2\pi}$. Show work, No calculator.

79. Solve the equation $\boldsymbol{2\sin^2\theta - 3\sin\theta - 2 = 0}$ on the interval $\boldsymbol{0 \le \theta < 2\pi}$. Show work, No calculator.

80. Establish the identity.

$$\frac{\tan u - 1}{\tan u + 1} = \frac{1 - \cot u}{1 + \cot u}$$

81. Find the exact value of the expression $\boldsymbol{\cos\frac{5\pi}{12}}$.

82. Use the given information given about the angle $\boldsymbol{\theta}$, $\boldsymbol{0 \le \theta \le 2\pi}$, to find the exact value of $\boldsymbol{\sin 2\theta}$. $\boldsymbol{\cos\theta = \frac{3}{5}, \frac{3\pi}{2} < \theta < 2\pi}$

83. Solve the problem. A building 200 feet tall casts a 90 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.)

84. Solve the triangle. $A = 40^o, B = 50^o, a = 1$. Show work.

85. Solve the triangle. $a = 6, b = 13, c = 15$. Show work.

86. SHOW WORK in finding EXACT(no decimals) values of each of the following: 7pts each a) $\sec\left[\sin^{-1}\left(\frac{-5}{13}\right)\right]$ b) $\cos\left[\tan^{-1}\left(\frac{4}{3}\right) - \sin^{-1}\left(\frac{-12}{13}\right)\right]$ c) $\csc\left[2\cos^{-1}\left(\frac{-7}{25}\right)\right]$

87. Solve the equations for the requested solution style, leaving all answers in fractions of π. 8pts each a) $4cos^2\theta = 3$ ALL solutions b) $\sec 2\theta = -\sqrt{2}$ on $[, 2\pi)$

88. Use an appropriate $\alpha \pm \beta$ formula to find an EXACT value of $\cos 105^o$ 12pts

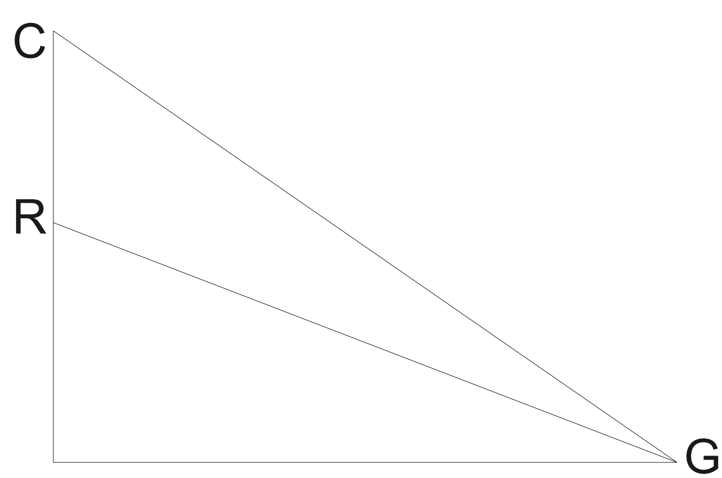
89. Copy the Identity into your booklet and prove it by working on ONE SIDE ONLY. 13pts

$$\frac{1}{\tan\theta - \sec\theta} + \frac{1}{\tan\theta + \sec\theta} = -2\tan\theta$$

90. Solve the Right triangle with: $b = 25.3, \quad a = 17.9, \quad C = 90^o$. Round to the nearest tenth. 10pts

91. Solve the triangle with side lengths of 15.6, 18.2, and 28.7, rounding to the nearest tenth. Also, find its Area. Round to the nearest tenth. 16 pts

    7. A cliff(point C) on a mountain is at an unknown height. One person hangs a rope of length 250 feet from the cliff down to point R. Another person on the ground(G) measures the angles of elevation to the rope at 18.2^o and to the cliff at 23.7^o. Find the height of the cliff to the nearest tenth of a foot. 12pts REDRAW THIS FIGURE IN YOUR BOOKLET AND LABEL ANY PARTS YOU USE.



92. For the point $(r, \theta) = (3, \dfrac{2\pi}{3})$, plot the point and then find other polar cordinates (r, θ) of the point for which (a) $r > 0, -2\pi \leq \theta < 0$ (b) $r > 0, 0 \leq \theta < 2\pi$ (c) $r < 0, 2\pi \leq \theta < 4\pi$

93. Find the rectangular cordinates for each point. (a) $(6, \dfrac{4\pi}{3})$ (b) $(-3, -346^o)$

94. Find the polar coordinates for each point. (a) $(-7, 3)$ (b) $(-3.4, -2.6)$

95. Write the equation $x^2 = 2y$ using polar coordinates.

96. Write the equation $r = 8\cos\theta$ using rectangular coordinates.

97. Write each complex number in polar form. Use degrees. (a) $-9 + 9i$ (b) $4 - 4\sqrt{3}i$

98. Write each complex number in rectangular form.
(a) $2(\cos 330^o + i \sin 330^o)$ (b)$0.3(\cos 240^o + i \sin 2340^o)$

99. Find $z \cdot w$ and z/w, and leave in polar form. $z = \cos 110^o + i \sin 110^o, \quad w = \cos 10^o + i \sin 10^o$

100. Write $100(\cos 80^o + i \sin 80^o)$ in standard form $a + bi$.

101. Find all of the complex cube roots of -4.

102. Find the position vector $\vec{v}$ with initial point $P = (4, 9)$ and terminal point $(4, 6)$.

103. Find $3\vec{v} - 2\vec{w}$ if $\vec{v} = 3\vec{i} - 7\vec{j}$ and $\vec{w} = -5\vec{i} + 7\vec{j}$.

104. Find $||\vec{v}||$ if $\vec{v} = -7\vec{i} - 6\vec{j}$

105. Find the unit vector in the same direction as $\vec{v} = 7\vec{i} - 4\vec{j}$.

106. Find the vector $\vec{v}$ whose magnitude is 7 and whose component in the $\vec{i}$ direction is both positive and equal to the component in the $\vec{j}$.

107. Find the vector $\vec{v}$ given that the magnitude is 7 and the angle it makes with the ositive x-axis is $\alpha = 225^o$.

108. Use the vectors $\vec{v} = 4\vec{i} + 8\vec{j}$ and $\vec{w} = -8\vec{i} + 4\vec{j}$ to answer the folowing questions.
(a) Find the dot product.
(b) Find the angle (in degrees) between $\vec{v}$ and $\vec{w}$.
(c) Determine whether the vectors are parallel, orthogonal, or neither.

109. Determine b so that vectors $\vec{v} = 3\vec{i} + \vec{j}$ and $\vec{w} = \vec{i} + b\vec{j}$ are orthogonal.

110. Use the vectors $\vec{v} = -7\vec{i} + 9\vec{j}$ and $\vec{w} = 5\vec{i} + 2\vec{j}$ to decompose $\vec{v}$ into two vectors $\vec{v}_1$ and $\vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{w}$ and $\vec{v}_2$ is orthogonal to $\vec{w}$.

111. Find the distance from $P_1(1, 2, 3)$ to $P_2(4, 5, 6)$.

112. Find the position vector for $P(2, -1, 3)$ and $Q(0, 3, -4)$

113. If $\vec{v} = 6\vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{w} = -\vec{i} + 5\vec{j} - 2\vec{k}$ find:
(a) $||\vec{v}||$ (b) $\vec{v} + \vec{w}$ (c) $\vec{v} - \vec{w}$ (d) $2\vec{v}$ (e) $2\vec{v} + 4\vec{w}$

114. Find the dot product for $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{w} = 5\vec{i} + 9\vec{k}$

115. find the angle between $\vec{v} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{w} = 4\vec{i} - \vec{k}$