First Hour Exam

- No Calculators, No Cell Phones
- Write clearly your name, the name of your TA, and the discussion time on the exam booklet.
- Show all work in exam booklet. Clearly label and box answers. If no work then no credit. Nothing written on the exam sheet will be marked.
- Turn in the exam booklet. You can keep the exam sheet.

(10 pts) **1.** Do the following formulas define functions of x (i.e. where x is the argument)?

(a) $y + x = 15$.

(b) $x^2 + 3y^2 = 25$.

(c) $x + y^2 = 7$.

(25 pts) **2.** Consider the polynomial function $p(x) = x^3 - 2x + 4$.

(a) How many complex zeros does this function have?

(b) Identify potential rational zeros and check if they are indeed zeros.

(c) Find all zeros of $p(x)$. Show all steps.

(d) Write down the function $p(x)$ as a product of linear factors.

(10 pts) **3.** Consider the functions $f(x) = \frac{2x}{x-1}$ and $g(x) = x^2$.

(a) Find the domains of $f(x)$ and $g(x)$.

(b) Find the domains of the composite functions $f \circ g(x)$ and $g \circ f(x)$.

(45 pts) **4.** Consider the function $f(x) = \frac{x^2+1}{x^2-4}$.

(a) Find the domain of this function.

(b) Verify that this function is even.

(c) Find x- and y-intercepts of $f(x)$.

(d) List vertical asymptotes. Determine the behavior of the function to the left and to the right of one of the vertical asymptotes by plugging in numbers. Use properties of the function to describe what happens to the left and to the right of another vertical asymptote.

(e) Determine the end behavior of the function as $x \to \infty$. You should find a horizontal asymptote. Determine if the function approaches the horizontal asymptote from above or below by plugging in a number. Based on your results above, describe the end behavior of $f(x)$ as $x \to -\infty$.

(f) Sketch the graph of the function, showing everything determined in above on the graph. Use at least $1/2$ of a page for your sketch.

(g) Using the graph, solve the rational inequality

$$\frac{x^2+1}{x^2-4} < 0.$$

Give the answer in the interval notation.

(10 pts) **5.** Write the rule of a function $g(x)$ obtained by performing the following transformations on $f(x) = x^2$.

(a) Stretch $f(x) = x^2$ vertically by a factor of 8.

(b) Shift the result of (a) horizontally by 2 units to the left.

(c) Shift the result of (b) vertically by 7 units down.